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Article

The Physics of Relationships

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Abstract

Reprint of the human chemistry, human thermodynamics themed article 'The Physics of Relationships', by American astrophysicist Christopher Hirata, consisting of five parts: (1) Thermochemical Approach to Relationships, (2) Complex Equilibria of Men and Women, (3) Reaction Kinetics, (4) Neutron Scattering: A Cautionary Tale, and (5) The Shell Model, the original article first published online in circa 2000, listed as a subpage of the Caltech Theoretical Astrophysics and Relativity Group (Tapir.Caltech.edu).

The Physics of Relationships

In the true spirit of Caltech (I'm not sure if this applies to Princeton), I devote this section of my website to the application of basic physical principles to relationships, particularly the romantic kind. Before I do this, I will make a few comments. You should understand them before you proceed. They aren't hard to understand:

Don't take it too seriously. This site is for your amusement, and it does not serve any other purpose. I am not a counselor, and if you have a real problem with your "significant other" then this page won't help you solve it. I take no responsibility for what you do with this information. Also recall that I don't have a girlfriend, so what do I know about this? Only what physics can tell me. [**UPDATE:** Since I wrote this page I have found Annika and we are engaged. This confirms that I know what I'm talking about.]
This site is geared primarily toward nerds. If you're not a nerd, this doesn't mean I want you to go away. In fact, it will give you some idea of the types of jokes frequently made in Caltech dorms. You may or may not find it interesting.
This is not a dirty site. For my younger audience: since this is the Internet, and I recognize that little kids have access to any material I put here, this site contains no sexually explicit material. (This is also because I don't want losers reading my page.)

So enjoy the compilation of worthless applications of physics and mathematics to relationships.

1. Thermochemical Approach to Relationships

For many of us, thermochemistry was our first science subject which involved nontrivial mathematics. It therefore seems appropriate to apply thermochemistry to relationships first before attempting something harder. My high school chemistry course spent about a month on the topic; yours probably did, too, in one or another way. But maybe it has been years since you last sat in a chemistry class learning about that wonderful topic; and so I will refresh your memory.

A general reversible chemical reaction is written as:

$$m_1 A_1 + m_2 A_2 + ... + m_p A_p \text{ <--> } n_1 B_1 + n_2 B_2 + ... + n_q B_q$$

and has equilibrium constant: (where the brackets [] denote concentration of a chemical in, say, atoms per cubic centimeter)

$$K = [B_1]^{n1} ... [B_q]^{nq} / [A_1]^{m1} ... [A_p]^{mp}$$

as well as having internal energy change DE, volume change DV, and entropy change DS^0 at standard concentrations. If the pressure is P and the temperature is T then the equilibrium constant can be found from:

$$-k_BT \ln K = DE + PDV - TDS^0$$

Here k_B is Boltzmann's constant and is roughly equal to $8.6x10^{-5}$ eV/K. I will leave it as an exercise to work out from what I have just told you such wonders as Le Chatlier's (did I spell that right?) principle: that if I make a change in the concentration of one substance, the reaction equilibrium will shift to compensate for the change.

Okay, okay, this isn't theoretical physics, we are trying to actually get results, not simply develop formalism. So I define the following systems, to be used throughout my notes on the the physics of relationships:

Number of people in a population = N;
Fraction of people who are girls = x;
Fraction of people who are boys = y;
Number of paired relationships = M;
Fraction of men who are free = f_y;
Fraction of women who are free = f_x.

Before examining the equilibrium constant, we note some *mathematical* facts which may be of importance. Specifically, we can say immediately that x+y=1 and that M<=max(Nx,Ny). Also we have formulas for $f_{x,y}$ since trivially f_x=1-M/Nx and analogously for the men, f_y=1-M/Ny. This allows us to obtain an important relation between f_x and f_y:

$$x[1-f_x] = M/N = y[1-f_y]$$

As an example, at Caltech we have N roughly equal to 900, with x=1/3 and y=2/3 (or so we are told -- THE RATIO R=y/x is taken to be equal to 2/1.) It is observed here that M ~ 200 yielding f_x=1/3 and f_y=2/3; one should not take too seriously the numerical coincidence that x ~ f_x and also that y ~ f_y, although as an exercise you may show that these two statements are equivalent.

Next we consider the reaction occurring, for after all, I opened this chapter by discussing chemical reactions and their equilibria, and thus you must be wondering where I will draw the connection. Well, the answer is very simple: we take boys and girls as the basic elements and symbolize a single girl as X and a single boy by Y. Note that capital letters are used to denote the boys and girls themselves, whereas lowercase letters denoted actual quantities. We note that the concentrations of X and Y, defined by the number of single girls and the number of single boys as fractions of the total population, are: [X]=xf_x and [Y]=yf_y.

(Additional note: some object to the use of the symbol Y because it is already taken for yttrium, element number 39 on the periodic table. Well guess what, yttrium is a stupid, dinky metal with only one stable isotope, ^{89}Y, whose primary use appears to be in the form of Y_2O_3 in color TV and computer monitor phosphors - it is used in the red phosphors. If you care sufficiently about the red phosphors, which you probably don't since you're looking at an almost red-free page (the cyan text contains only a tinge of red), that's tough. Now let's get on with it.)

So the simplest reaction, and the only one we are currently considering right now, is the reaction:

$$X + Y \text{ <--> } XY$$

There are many other compounds that can be formed in addition to the pairing of men and women; but these are rare and non-traditional. Or maybe they're not, but we won't consider them yet due to the mathematical mess they would cause. Such compounds include the gay molecule Y_2, the lesbian molecule X_2, and the middle-Eastern polygamous molecule X_4Y. But let us concern ourselves with the above reaction; its equilibrium constant is:

$$\begin{aligned} K &= [XY] / [X][Y] \\ &= (M/N) / (xf_x * yf_y) \\ &= (M/N) / ((x - M/N) * (y-M/N)) \\ &= MN / (Nx-M)(Ny-M) \end{aligned}$$

For example, at Caltech K=4.5. Using the theory that the equilibrium constant is independent of concentrations and is a function only of temperature and pressure, we may determine what f_x and f_y would be if Caltech had equal numbers of boys and girls. Setting x=y=1/2, our equilibrium constant expression becomes:

$$K = MN / (N/2-M)(N/2-M) = MN / (N/2-M)^2$$

$$N^2/4 - MN + M^2 = MN/K$$

$$M = (^N/_2) [(1+K^{-1}) - [(1+K^{-1})^2-1]^{1/2}]$$

$$M = (^N/_2) [(1+K^{-1}) - K^{-1/2}(2+K^{-1})^{1/2}]$$

For K=4.5 this yields M=0.26N, or $f_x=f_y=0.48$. So listen up Caltech guys; whereas 67% of you are free right now, we just showed that evening out the ratio reduces that only to 48%, and so nearly half of you still won't have girlfriends. Well, it's an amazing revelation. Now for the amazing revelation for my female readers: 33% of you (at least those of you at Caltech) are currently free (that is, single); and that would increase to 48%. Of those of you who have boyfriends, 23% would lose them. You don't want more girls around if the chances are that high!

Now let us go the other way. Suppose we turn time back at Caltech to some number of years ago, when we had x<<1. Then our equilibrium constant expression gives:

$$K = MN / (Nx-M)(Ny-M)$$
$$\text{approx.} = MN / (Nx-M)N = M/(Nx-M)$$

$$K(Nx-M) = M$$

$$Nx = M (1+K^{-1})$$

$$M = Nx/(1+K^{-1}) = KNx/(1+K)$$

In other words, $f_x = 1 - M/Nx$ is now well approximated by:

$$f_x = 1 - K/(1+K) = 1/(1+K)$$

So then, for Caltech's K=4.5, we would have that 18% of the girls would remain single no matter how few there were. If one looks at the men, however, one sees a different pattern:

$$f_y = 1 - M/Ny = 1 - M/N$$
$$= 1 - Kx/(1+K)$$

so that as x --> 0, f_y --> 1. What a surprise, when there are no girls, almost none of the guys are going out with girls! Needless to say, those guys will be pressuring that 18% of the girls pretty hard, but I leave for the psychologists the question of what exactly would happen; let me simply observe that x=0 may be a preferable situation to x=0.02.

There are a few other cases to consider. Suppose, for example, that we decided to make K very small, perhaps by giving our entire population enough marijuana to cause them to lose interest in relationships. (Don't try this experiment. It will suck if you do.) Then we see than M is negligible compared to Nx and Ny, and that:

$$M = NKxy$$

$$f_x = 1 - M/Nx = 1 - Ky$$

similarly

$$f_y = 1 - Kx$$

In this case, as K --> 0, there are no couples, again not surprising. Why should we get something surprising in these limits, anyway?

Next we may consider the opposite case, that of large K. In this case, the problem becomes similar to that of an acid-base equilibrium (in chemistry) or an electron-hole equilibrium (in semiconductor physics). These cases, and the role of polygamy and homosexuality, will be examined further in the next chapter.

Exercises.

1. Solve the equilibrium constant expression for M, f_x, and f_y in the general case.

2. If heat is released when a boy and girl get together, and they each take up the heat when they separate, then what is the sign of DE for the reaction X + Y <--> XY? Does K go up or down if the temperature rises, say, because the population takes a trip to a beach in Hawaii? Does this make sense?

3. Test out Le Chatlier's principle on the X + Y <--> XY equilibrium and check that your results make sense.

4. (challenge problem) Work out the mathematics of homosexuality in a men-only society through the reaction 2 Y <--> Y_2, considering limiting cases as was done in this chapter's treatment of heterosexual relations with both men and women present.

2. Complex Equilibria of Men and Women

In the last section we took a simple and brief look at relationships in terms of thermochemistry. Now we consider more complicated systems. The first type we will look at is an equilibrium of the two strongly interacting chemical species, female (X) and male (Y). Note that in this context, "strongly interacting" refers to the strength of the interaction, that is, the reaction:

$$X + Y \text{ <--> } XY$$

runs nearly to completion in almost all cases, and also the concentration of XY is nearly constant in the given situation and is much greater than that of either X or Y. It does *not* refer to the nature of the interaction itself, which we do not yet consider; specifically a strong interaction need not involve the colour force or the exchange of gluons.

So where do strong interaction equilibria occur? They are common in nature; we can list here only a few of them. These will, however, serve our purpose in the sense of providing examples of successful theories of strong interaction equilibria upon which we may base the theory of strong relationships. Those we choose to focus on are:

Acid-base equilibria between the H_3O^+ and OH^- ions in aqueous solution, which form the basis for much of our body chemistry (including that responsible for the X + Y <--> XY reactions in the first place);
Electron-hole equilibria in semiconductors such as Si, Ge, or GaAs which form the basis for many modern electronic devices such as light-emitting diodes (LEDs), transistors, operational amplifiers ("op amps") and computers;
Electron-positron equilibria and other particle-antiparticle equilibria important at high temperature, for example in the cores of old massive stars or in the first minutes after the Big Bang.

We will model our notation after that of acid-base chemistry, as this is the subject which has most closely paralleled the course which we intend to traverse; and, for lack of better choice, we shall treat the girls X as the acidic and the boys Y as the alkaline component. That is we draw the parallel:

Acid-base equilibrium variable	Relationship variable
$[H_3O^+]$	[X]
$[OH^-]$	[Y]
10	e
$pH = - \ln[H_3O^+] / \ln 10$	$\$X = -\ln[X]$
$pOH = - \ln[OH^-] / \ln 10$	$\$X = -\ln[Y]$

Notice here that while the pH is defined to the *base ten* system, the corresponding relationship system is defined according to the natural (analytic) base e=2.71828... due to its greater utility in describing certain processes. This difference between the two systems is emphasized in the third line of the table and allows us to avoid a pitfall that has ruined the fields of acid-base and electro-chemistry.

The first key idea is that of a *dissociation equilibrium constant*, denoted by K_w in acid-base chemistry and corresponding to:

$$K_w = [H_3O^+] [OH^-]$$

where the "w" subscript represents water. We define for ourselves a similar K_c ("c" for couples) where:

$$K_c = [X] [Y]$$

In the regime in which we are interested the XY compound is dominant. There will thus be roughly 1 XY for every two people so $[XY] \sim {}^1/_2$. In this case we have that the XY formation equilibrium constant K was given by:

$$K = [XY] / [X][Y] = 1/(2[X][Y])$$

or

$$K=1/(2K_c)$$

We have thus established a critical relation between the relationship's dissociation and formation constants: that their product KK_c is equal to 1/2. Although this seems like a fair amount of progress, it is nevertheless tautological and devoid of content. We remedy this by plugging in numbers. As argued in chapter 1, for Caltech we have K = 4.5 and hence Kc = 0.11. We may even define a *logarithmic* constant $\$K_c = - \ln K_c$; this would in this case be equal to 2.2. Note that in acid-base chemistry the equivalent pK_w is 14.

Now we are ready to consider the case of strong acids and strong bases. In acid-base chemistry, we learned what these are: the strong acids are HCl, HBr, HI, H_2SO_4, HNO_3, and $HClO_4$. The strong bases are the oxides, hydroxides, and sulfides of the elements Li, Na, K, Rb, Cs, Ca, Sr, and Ba. In electron-hole equilibria in Si these roles are played by B, Al, and P; in Ge they are played by Ga or As; and so on, I think you get the point. Here there is no requirement for a strong acid to have an anion or a strong base to have a metal; rather we take free girls to be the strong acids and free boys to be the strong bases. A strong acid X dumped into (nearly) pure XY, if given initial concentration X_0, will yield $\$X = - \ln X_0$; a strong base Y dumped into an identical starting solution and given initial concentration Y_0 will yield $[X] = K_c/Y_0$ and hence $\$X = \$K_c + \ln Y_0$. An equilibrium solution of equal numbers of boys and girls yields $\$X = ({}^1/_2)\ \$K_c$; the corresponding "neutral" pH in acid-base chemistry is ${}^1/_2$ of 14 or 7. Doesn't it all make sense?

As a concrete example, consider a mixture of 60% boys and 40% girls with strong interaction and the Caltech equilibrium constant $\$K_c = 2.2$. What fraction of the girls will be free? We could solve this by the standard method of setting up a nasty quadratic equation (*see* Exercise 3.) We may note that we have an excess of boys by Y_0 = 20% = 0.2 so that \$X = \$$K_c$ + ln Y_0 = 2.2 + ln 0.2 = 0.6. Note that this isn't a particularly good \$X value since it is less than 1.1 = \$$K_c$ / 2 -- thus we have not sufficiently saturated the equilibrium for our formula to be valid. It is important to watch out for these kinds of difficulties. But if we proceed anyway, the free girl concentration [X] is 0.55, a contradiction. This is caused, of course, by our approximation that \$$K_c$ is large. If \$$K_c$ were 10, then the theory would work much better; the reader may verify this.

(Note: most authors only give examples of their theory working. I am sufficiently honest to show you the limitations of my theory, perhaps as a cautionary note. Life operates to higher than first order.)

So up until now we have considered only a limited set of arrangements. Now let us add in *weak acids* and *weak bases*. A weak acid in this regard consists of a somewhat horny girl paired to an unattractive guy; and a weak base is the reverse: a testosterone-crazed male who finds himself dating a girl who (according to Massey) "is a hundred pounds heavier than he is." We focus on the theory of weak acids; the weak bases are similar.

A generic weak acid will be denoted as XYa, where the new element Ya represents an unattractive male (or at least a male who will be dumped if there are other choices.) There is, of course, a corresponding element Xa which is the corresponding type of girl. So a weak base would be denoted XaY. We study the disocciation of XYa in a medium consisting primarily of paired couples XY, that is, we are interested in the reactions:

XYa <--> X + Ya

and

XaY <--> Xa + Y

These have the equilibrium constants:

K_x = [X][Ya]/[XYa]

and

K_y = [Xa][Y]/[XaY]

In cases of a titration of solutions of XYa with a strong base (Y), we can make some definitive conclusions regarding the nature of the new beast Ya. Recall that a titration in this particular example consists of taking a bunch of couples, lightly mixed with some normal girls and unattractive (?) boys, and then adding some of the more attractive guys and seeing what happens to the proportion of free girls. (Note that this is easier for me, as a male, to stomach than the reverse concept, that of a bunch of guys ditching their girlfriends for nicer-looking material. Yuck.) The key to understanding such a titration is to define a new parameter, \$$K_x$, by:

\$$K_x$ = - ln K_x

and then noting that we have the interesting relation for \$X:

\$X = - ln [X] = - ln ([X]*[Ya]/[XYa]) + ln ([Ya]/[XYa])

= \$$K_x$ + ln ([Ya]/[XYa])

Those familiar with acid-base chemistry may recognize this equation.

There are two additional concepts of note here: hydrolysis and polyprotic acids/bases. Hydrolysis is the reaction of an "ion" (not X or Y) with "water" (couple XY) as follows (for example):

Xa + XY <--> XaY + X

Note that this reaction proceeds very weakly since the male Y prefers to be with X than Xa. However it does occur to a limited extent and its equilibrium constant (see Exercise 5) is nonzero. Studies of hydrolysis are useful in determining \$X at the equivalence point of a titration.

Finally, we note that we have yet to break the symmetry between men and women, that is, we have a binary ($\mathbf{Z}_2$) symmetry group with two elements, the identity E and the male-female reversing operation s. This situation can be remedied by allowing the men only to be polygamous; this is not my original idea, it has existed for some time in many cultures. (I am not a believer in polygamy, by the way ...) For example, in Muslim culture it is acceptable under certain conditions to form the X_2Y, X_3Y, and X_4Y compounds. At each step there is an equilibrium constant:

XY + X <--> X_2Y
K_2=[X_2Y]/[XY][X]

X_2Y + X <--> X_3Y
K_3=[X_3Y]/[X_2Y][X]

X_3Y + X <--> X_4Y
K_4=[X_4Y]/[X_3Y][X]

A full analysis of this system is sufficiently complicated that it is left as an exercise to the reader.

Exercises.

1. Research and explain, in no more than a paragraph, why chemists of times past were so idiotic as to use the base ten rather than the natural logarithmic scale.

2. Why would Caltech not be a good model for a strong interaction equilibrium of X, Y, and XY? Can you remedy the situation, perhaps by developing a theory in which [X]<<1?

3. Suppose there are 40% girls and 60% boys in a population with $\$K_c$=2.2. What fraction of the girls will be free according to the "exact" equilibrium constant? How bad or good is the theory of this chapter in explaining this system?

4. Verify that the acid-base equilibrium theory works best at large $\$K_c$.

5. Find the equilibrium constant of the following reaction in terms of $\$K_y$ and $\$K_c$:

$$Xa + XY <--> XaY + X$$

6. Work out the character table for the group $\mathbf{Z}_2$ which has two elements. Does this shed any light on relationships? Why or why not?

7. Use the equations for the polygamous men to find the equilibrium constant of the reaction:

$$Y + 4 X <--> X_4Y$$

8. Using techniques similar to those here, examine the role of male homosexuality by defining an appropriate equilibrium constant. Does male homosexuality increase or decrease the number of free women? Does this make sense?

3. Reaction Kinetics

Life does not exist as an equilibrium, as we all know. We are born, live our lives as best we can, and eventually pass on to whatever comes next. Meanwhile, relationships are formed and break up. In determining this fact for chemical reactions, scientists once resorted to isotopic labeling of compounds; we don't need any isotopes here, however, since it is plainly obvious that many boys go through a dozen or more girls before settling down, and many girls behave correspondingly.

To begin our approach to kinetics, we take some cues from chemistry. Kinetics are important not only for their own purposes, but also because they help us understand the structure and mechanisms of relationships. Thus perhaps we should think about possible mechanisms of reaction kinetics when constructing a formula. I begin by suggesting the following relation:

$$d[XY]/dt = \sum_{i=0,1,2...} \sum_{j=0,1,2...} \sum_{k=0,1,2...} (c_{ijk} [XY]^i [X]^j [Y]^k / i!j!k!)$$

You will recognize that this is nothing other than a Taylor series in [XY], [X], and [Y]. The coefficients c_{ijk} are the partial derivatives and cross-derivatives of d[XY]/dt:

$$c_{ijk} = (d/d[XY])^i (d/d[X])^j (d/d[Y])^k (d[XY]/dt) |_{[XY]=[X]=[Y]=0}$$

Note that I have very craftily assumed a lot of cool nice continuity and differentiability properties that I could never hope to prove by the standards of real analysis. But anyway, this is real life, there are no ugly functions here -- relationships are well behaved, didn't you know? So anyway, we proceed to physically interpret some of the constants c_{ijk}. We will do this in table form for some mysterious reason (I think it is easier to follow):

Const.	Interpretation
c_{000}	This must be zero since if there's nobody around you won't find too many relationships, and the derivative of zero is zero. You can't get something for nothing. Sorry, that's true, even love obeys conservation laws.
c_{100}	This is the self-dissociation constant for relationships. Note that if there is only XY around, this constant tells us how quickly it falls apart. By conservation of men and women, c_{100} cannot be positive; that is, relationships can fall apart, but they can't spring out of nowhere in the absence of people to form relationships from. The mean life of a relationship is roughly $-1/c_{100}$; notice that at Caltech, therefore, where a relationship may typically last six months, $c_{100} \sim 2\ yr^{-1}$.
c_{010}	This must be zero since a girl can't date a nonexistant boy (note that the derivative for c_{010} does not involve anything other than X. It's all zero, folks.)
c_{001}	If you understood why c_{010} was zero, you can figure out this one, too.
c_{011}	This is the coefficient upon which many a cheesy Hollywood movie has been based. It refers to the spontaneous combination rate of X and Y which occurs in the absence of any external influences.
c_{111}	This describes the process in which a couple decides that some guy and girl they know ought to get together. They introduce Him and Her and the two hit if off real well. This is a low success probability process, so in most circumstances it is safe to take $c_{111} \sim 0$.
c_{110}	In this event, *she* induces a couple to break up.
c_{132}	I'm a physicist, in my world higher order terms are inherently negligible.

In case the pattern isn't clear yet: the c_{ijk} coefficient corresponds to processes involving i couples, j free girls, and k free boys.

So now let us connect this to the old equilibrium theory. Suppose that we neglect all but the c_{100} and c_{011} terms -- that is, the terms corresponding to the intrinsic unstimulated dissociation of XY and the direct uncatalyzed X + Y --> XY reaction. In this case, we have the relation for d[XY]/dt:

$$d[XY]/dt = c_{100}[XY] + c_{011}[X][Y]$$

Setting d[XY]/dt to zero (appropriate in equilibrium) yields a miraculous result:

$$K = [XY]/[X][Y] = -c_{011}/c_{100}$$

So this is indeed predicted to be constant! This result, combined with our estimation of c_{100} above allows us to determine c_{011}:

> K = 4.5;
> $c_{100} = -2\ yr^{-1}$;
> $c_{011} = 9\ yr^{-1}$.

Now let us apply these results to a practical problem: how long will it take poor Chris Hirata to get a girlfriend? We can see that the rate R at which XY is forming from X and Y (note R is not equal to d[XY]/dt because the latter takes into account dissociation as well) is given by $R=c_{011}[X][Y]$. We are interested in the supply of men [Y] divided by the rate R at which they are being introduced into relationships ... this would be:

$$[Y]/R = 1/(c_{011}[X])$$

At Caltech we have [X] = 0.11 and $c_{011} = 9\ yr^{-1}$. Multiplying and inverting and so on yields 1.0 years. This is the mean time for me to find a girlfriend ... and that may suck because I'm only going to be at Caltech for another year. It sort of makes me want to cry. Oh well, I'm trying to teach you about life, I'm not here to whine. (Update: Chris graduated from Caltech June 15, 2001 without a girlfriend.)

The exact solution to the differential equation for d[XY]/dt is considered in the exercises. For now, we concern ourselves with an approximate solution in terms of the *relaxation time* T. T can be calculated by taking d[XY]/dt and considering its value if we alter the concentrations from their equilibrium positions slightly:

$$[XY] = [XY]_{eq} + d$$
$$[X] = [X]_{eq} - d$$
$$[Y] = [Y]_{eq} - d$$

The net rate d[XY]/dt can be expanded as a Taylor series in d. Note that the zeroeth-order term vanishes since there is no net rate in equilibrium. We have: (neglecting second-order terms in d because they are of course small)

$$dd/dt = d[XY]/dt$$
$$= c_{100}[XY] + c_{011}[X][Y]$$
$$= c_{100}d - c_{011}([X]_{eq}+[Y]_{eq})d$$
$$= [c_{100} - c_{011}\ ([X]+[Y])_{eq}]\ d$$

This yields a relaxation time, or time for d to decrease by a factor of e, of:

$$T = 1 / [- c_{100} + c_{011}\ ([X]+[Y])_{eq}]$$

For example, at Caltech, T is equal to 0.2 years. This is the timescale on which the dating relationships equilibriate according to the numbers of boys and girls present.

Exercises.

1. Explain why d[XY]/dt cannot be negative if [XY]=0.

2. Why is c_{110} negative?

3. Prove Stirling's formula:

$$n! \sim (2p)^{1/2} * n^{n+1/2}e^{-n}$$

and use it to justify my hasty assertion that the denominators i!j!k! go to infinity rapidly with increasing i,j,k.

4. Assume only c_{100} and c_{011} terms are relevant in the differential equation for d[XY]/dt. Solve for [XY] as a function of time and initial conditions.

5. Work out a formula for the mean time for a girl to find a boyfriend in terms of the mean lifetime of relationships and the equilibrium constant K, assuming only the c_{100} and c_{011} terms are significant.

6. Repeat this derivation of kinetics for the case of a society of homosexual men.

4. Neutron Scattering: A Cautionary Tale

Now we begin to consider the nature of the forces involved in the relationships we are studying. This is not an easy subject. There are numerous pitfalls in applying what we know about ordinary human behavior to love, or even to sexual desires; and so to reinforce this fact I provide you with the following to consider: these processes occur on human time and length scales which are well known to us, and yet extrapolating (or interpolating?) from the known into the unknown may be dangerous. Here we compare sexual relations to neutron scattering in order to see what conclusions we may derive. We eventually determine that *resonance* may be important in the microscopic non-statistical approach to relationships. (Note: what follows is adapted from "Hot Sex, Cold Neutrons" by Chris Hirata.)

So one asks, what is the nature of sex and love and so on, and their relation to the other everyday experiences around us? And the answer, of course, follows only from an observation of this world and the processes that occur in it.

It must be noted, if it is not obvious, that I have never experienced any sort of romance. It is also true that I never will, and yet my curiosity drives me to inquire as to the nature of the experience. And it is true that physical theory is the only way in which to answer this question, other than direct experience. And while direct experience is undeniably a more reliable way to answer such questions, a few major problems with direct experience must be addressed: first, that it is not going to happen; and second, that various other people whom I know have been involved in romances, and in some cases they have been involved in many consecutive romances, but have failed to take data during those treasured minutes of sex. Love lasts longer but they don't want to analyze their relationships for fear of destroying them. More seriously, they do not take time to step back and make objective and scientific observations. And thus, for all practical purposes, the mysteries of sex and love can for the time being only be revealed by theory.

It is, first of all, clear that direct derivation of the properties of our relationships from the laws of quantum electrodynamics is not feasible with our current computational capabilities. The Lagrangian for quantum electrodynamics is:

$$L = (-^1/_4)F_{mn}F^{mn} + y^+g^0(ig^mD_m\text{-}m)y$$

so we probably can't solve this for ourselves. Even non-relativistically, when the all-important correlation effects are considered, the human body is described by a wave function on a space with trillions of trillions of dimensions; and given this intractable complexity, what hope is there of unraveling the problem of two interacting people? No, if there is any hope of theoretically solving the problem of sex, a higher-level model of the system is necessary.

In the absence of a better way to proceed, we choose such a higher-level model by first considering nuclear physics, the most elementary discipline which has the three characteristics of a good analogue to the relationship problem. First, there is an underlying theory, in this case, a combination of quantum electrodynamics and quantum chromodynamics, which cannot in any practical way be solved without serious limitations in the form of simplifying assumptions; second, there is some hope of deriving, either from first principles or from empirical observation, a set of rules which can guide us along the path to truth; and finally, unlike love, there exists a mountain of hard experimental data, much of it unclassified, which enables us to judge the effectiveness of extrapolating a theory into the unknown.

So suppose we are studying nuclear physics. Here we restrict ourselves to the study of reactions of neutrons with atomic nuclei, a set of processes with which I am all too familiar. Suppose we choose to study the simplest of all nuclei, the proton, at least initially. We learn in our experiments at moderate energies, from perhaps an electron volt (note: 1 electron volt is $1.6\text{x}10^{-19}$ joules) to a thousand electron volts, that the cross section for reactions is constant at twenty barns, where a barn, for those readers who are not familiar with nuclear physics, is one trillionth of one trillionth of a square centimeter. And if we take as our model a constant cross section, have we done justice to our problem? In our energy range we have; but there are problems. As we push to higher energies, we find that the cross section begins to decrease. At a million electron volts it is only five barns. What has happened? We pushed outside of the range of the known into the unknown. And when one pushes beyond the database of points upon which a theory is constructed, one should expect to be surprised.

But based on the new data we formulate a new theory, in which the scattering amplitude is the Fourier transform of a Yukawa potential:

$$V(\mathbf{r}) = A\ r^{-1}\ e^{-mr}$$

$$ds/dW = c\ \left| \langle \mathbf{p}_{out} \mid V(\mathbf{r}) \mid \mathbf{p}_{in} \rangle \right|^2$$

where m is a constant.

The new theory holds up quite well even into the millions of electron volts. But suppose we push the other direction. At a ten-thousandth of an electron volt, we find a cross section of twenty-six barns. Obviously something else has gone wrong: there is a new process, deuteration, that we had not conceived of. (Technical note: deuteration in this case corresponds to the reaction:

$$^1H + n \text{ --> } ^2H + g$$

where the gamma ray emerges with ~ 2224 keV energy.) Once again we pushed outside of the range of the known into the unknown, and when one pushes beyond the database of points upon which a theory is constructed, one should expect to be surprised.

But when we studied neutron reactions well over a particular energy range, there were no surprises within that range, even though we did not sample each of the uncountably many energies in the range. And so we return to relationships: do the well-studied processes in life, from washing cars to doing homework to playing basketball to climbing stairs to programming a computer, adequately cover the range of physical quantities in love, sex, and so on? We see that the answer seems to be a very definitive yes.

In the range of physical dimension of geometrical length, mass, and power consumption by the body, sex resembles, at least to an order of magnitude, any process of human life, including the ones described in the previous paragraph. Specifically, geometrical lengths are of order a meter, masses of order tens to a hundred kilograms, and power consumption by the body is of order a hundred watts. In the area of time, sex is reported to take a thousand seconds or possibly greater than this amount by a factor of order unity. Many human activities with which I am very familiar consume this amount of time, to an order of magnitude, such as going to lectures, delivering lectures myself, solving a physics or mathematics problem, going for a walk, or eating a moderately sized meal. Relative velocities of ones to tens of centimeters per second, as occur in sex, are bracketed from below by sleeping or sitting and from above by running. Magnetic field strengths are presumable of order a third of a gauss whether there is sex or not. And as for love? A person in love and a person out of love cannot easily be distinguished by measuring their heights, weights, electric quadrupole moments, and other physical parameters.

So we may justifiably conclude that the object of our examination is not so vastly different from other activities that it should not be adequately described by an extension thereof. And therefore it is all rather easy to describe; but there is, at least culturally, a mysticism surrounding the ordeal (?) of falling in love, having sex, having kids, raising kids, and so on. How can this "extraordinary" nature of love be reconciled with our observation that it operates at the same level as other, more ordinary processes?

For the answer we turn once again to neutron interactions. Consider this time not the almost trivial proton but the nucleus of a natural cobalt atom, ^{59}Co, which contains no fewer than twenty-seven protons and thirty-two neutrons locked together. A neutron incident upon the cobalt nucleus has some cross section for interaction; and it is this cross section, as a function of neutron energy, which we seek to determine.

At energies of one-half, one, three, ten, twenty-five, fifty, two hundred and fifty, and five hundred electron volts, our neutrons find cross sections of ones to tens of barns, with a general decrease toward the higher energies, roughly corresponding to a power law relating cross section (s) to energy (E):

$$s \sim E^{-1/2}$$

And thus we might conclude, on the basis of the preceding reasoning, that at an energy of one hundred and thirty electron volts, the neutron should have a cross section in this general range.

If you have a neutron source available, I suggest you measure the cross section at one hundred and thirty electron volts. For the rest of you, I shall not delay my statement until you do the experiment, but rather tell you that the experiment has been done. The cross section is ten thousand barns. In fact, near 130 eV, the neutron cross section of ^{59}Co can be approximated as a Cauchy curve:

$$s = 10^4 \text{ barns} / [\, 1 + ((E - 130 \text{ eV}) / 10 \text{ eV})^2]$$

Here is irrefutable proof that undiscovered phenomena can lie well inside the range of observations which we have taken. If you wish for other examples, consider that the thermal neutron cross sections of the first twenty elements are all no more than tens of barns; except for boron, which has a cross section of seven hundred barns. Or consider the asteroid belt, in which there exist the Kirkwood Gaps, regions nearly devoid of asteroids for no reason other than a resonance with the orbital motion of the planet Jupiter. Just because we've seen a lot doesn't mean we've seen it all, even in our own neighborhood.

And so one asks why these phenomena occur. In all the cases mentioned above, and nearly all the cases known in physics, the answer can be contained in one word: resonance. Resonant effects are the reason why the corner of phase space we think we understand may have something hidden in it that we don't know about. Resonance is the key not only to the collapse of the Tacoma Narrows Bridge, but to some of humanity's most ancient emotions as well.

So love is a resonant effect, we have learned; and so be it, for truth is always superior to imagination. And as with all resonances, love must have a resonant width, which would characteristically be denoted g. And the longer it is to last, the smaller the value of g, and the harder it is to find. And this may be the greatest lesson that modern physics can teach us about our desires.

Exercises.

1. Explain the -1/2 power law dependence of neutron cross section on energy at low energies and the 0 power law dependence (s = const) at medium energies. (*Hint:* Compare the de Broglie wavelength of the neutron to the typical size of an atomic nucleus.)

2. List three additional examples of resonances in which one could take data at the length, time, etc. scales of the resonance without realizing it was there.

3. (**Challenge Problem.**) Explain how the reaction rate for formation of XY (*see* Chapter 3):

$$R = c_{011}[X][Y]$$

can be made consistent with the concept of a resonance allowing X and Y to bind to each other. If there is one such resonance, what temperature dependence of c_{011} is anticipated?

5. The Shell Model

Now we consider the quantum basis for two of the more disgusting structures we wish to examine: polygamous systems and orgies. These will be examined using three-dimensional quantum mechanics, since even if "love is another dimension," there is clearly insufficient love between any two components of the system - okay, okay, I'd better not go further.

First consider a polygamous male who can have Z females. I do not know the origin of the letter Z, only that it is typically used to refer to atomic number in the theory of atomic structure. Well guess what, these guys who act as nuclei attracting a bunch of female electrons seem to have an "atomic number" Z > 1.

So in order to examine this quantum mechanically, we need a Hamiltonian for the system. The Hamiltonian for an atom, neglecting electron and nuclear spin effects as well as nuclear motion, is:

$$H = S_{i=1...Z} (p_i^2/2m - Ze^2/r_i + S_{j=1...i-1} e^2/r_{ij})$$

Before applying this Hamiltonian to polygamous relationships, we pause to examine its validity. The neglect of nuclear motion is probably valid since these types of guys tend to be lazy. We've also treated these guys as isotropic. This is the case since they are equally disgusting in all directions. Finally, the 1/r potential appears because this is in **3-D** and so only inverse and constant potentials satisfy the homogeneous Poisson condition. The constant term doesn't matter since it's a constant in the Hamiltonian. The coupling constant e^2, also known as the fine-structure constant and equal to ~ 1/137 in atomic structure is probably pretty small here since he isn't particularly attached to any of his women.

Now we examine the results from this Hamiltonian. Spherical symmetry (if we treat the cross-terms as having no angular dependence, a rough approximation) yields that the wave functions of the women will fall into the s, p, d, and f categories familiar from chemistry. More technically, if the rotation group SO(3) has generators J_i, then $[H,J_i] = 0$ and so there exists a complete set of energy eigenstates which are also eigenstates of J_z and J^2. The eigenstates have eigenvalues:

$$J^2 \Rightarrow (h/2p)(j(j+1))^{1/2},\ j=0,1,2,...$$
$$J_z \Rightarrow (h/2p)m,\ m=-j, ..., +j$$

which gives us a 2j+1 degeneracy for each of these levels. In fact, due to the spin 1/2 of a woman (you have to turn her around twice for her to look the same) the degeneracy is 2(2j+1). I would say more about this except I don't really want to talk about the basis functions in the 2 x j representation of the Lie group SU(2) x SO(3).

So we label the states with j=0 as "s" states, those with j=1 as "p", and those with higher j as "d" (j=2), "f" (j=3), and so on. If you're interested, these strange letters come from the days of Na (Z=11) spectroscopy where the lines were categorized as "sharp", "principal", "diffuse", and "fundamental". If I ever have a girlfriend, she will live in the lowest-energy sharp state, configuration $1s^1$ - no, this is not an advertisement.

So we find that there is a whole structure of states: the lowest energy level (n=1) has only an s state and thus can hold two girls. The next highest energy (n=2) has an s and a p (eight slots), the n=3 level has an s,p, and d (18 slots) and so on. In X-ray spectroscopy, where I once did some work, we call the n=1 level the K shell, n=2 the L shell, n=3 the M shell, and so on. In fact, because the L shell is not fully degenerate, we call the 2s states L_I, and the 2p states L_{II} or L_{III} depending on the alignment of the spin and angular momentum.

So, just as in chemistry, we have the noble gas configurations, He, Ne, Ar, Kr, Xe, and Rn. We thus expect polygamy to be especially stable when "he" has 2, 10, 18, 36, 54, or 86 girls. The fact that one-man, eighty-six woman relationships don't seem to happen that much can be attributed to the low abundance of higher-Z nuclei, or perhaps to higher order perturbative effects.

As an example, let us consider the woman configuration of a man with four wives. Filling in orbitals from the bottom up, we conclude that the configuration is $1s^2 2s^2$. Experimental verification of this configuration may be obtained by one of the following methods:

removing a woman from the picture and determining the amount of work done on her in order to get her to loosen her grip on that guy;
removing a woman from the picture and studying the characteristic X-rays emitted;
exciting some of the women to higher states with a laser;
taking an X-ray spectrum and looking at the K, L, etc. edges in the photoelectric absorption cross section;
taking a woman to a tanning salon and determining the UV wavelength necessary for ionization;
firing high-energy charged particles (e.g. women, men) into the situation and measuring the differential scattering cross section;
attempting Mossbauer spectroscopy and looking at the isomeric shift. (Note: messing with nuclear reactions might make Him mad.)

As a public service for the confused, I figured I would list some concrete examples of what I'm talking about. Well, here are some examples of men of different types and their electronic configurations:

Atomic number/Element	**Electron configuration**	**Examples**
H^+ (Z=1) hydrogen ion	not applicable	me, Mike Massey (hint: electron welcome)
H (Z=1) hydrogen	$1s^1$	my happily married father
He (Z=2) helium	$1s^2$	those guys who cheat
He* (Z=2) excited helium	$1s^1 2s^1$ and variants	those guys who cheat and are about to dump one of their honeys

Li (Z=3) lithium	$1s^2 2s^1$	if the guy is really crafty he might be able to do this. note that he can't be really close to all three women
Be (Z=4) beryllium	$1s^2 2s^2$	a guy who really wants a lot of wives but is constrained by the Islamic religion
B (Z=5) boron	$1s^2 2s^2 2p^1$	same guy as for Be except that he figures he can get away with 5 if the Qaran says 4 ... note that the last lady must move around him with nonzero angular momentum.
C^{2+} (Z=6) carbon ion	$1s^2 2s^2$	same guy as for Be except perhaps a little more accurate if he's still looking for women
Al^{3+} (Z=13) aluminum ion	$1s^1 2s^2 2p^6$	a typical Senator or Representative: got ten women, still looking for more
Ti^{4+} (Z=22) titanium ion	$1s^1 2s^2 2p^6 3s^2 3p^6$	another Senator or Representative; typical if you only consider Democrats
Hg (Z=80) mercury	$1s^1 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6$ $4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 6s^2$	Bill Clinton
Pu (Z=94) plutonium	$1s^1 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14}$ $5s^2 5p^6 5d^{10} 5f^6 6s^2 6p^6 7s^2$	your favorite Kennedy; note that plutonium has a relatively short half life, and can be destroyed on contact with small but fast-moving projectiles (neutrons)
Mt (Z=109) meitnerium	$1s^1 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14}$ $5s^2 5p^6 5d^{10} 5f^{14} 6s^2 6p^6 6d^7 7s^2$	as we explore further on the periodic table, and our culture becomes more and more accepting of non-traditional social behavior, what unknown phenomena will occur?

So as you can see, there is plenty of capability to examine polygamous systems. Go experiment and check my theory! (Also note: some women will have many men. The situation looks surprisingly similar. Call it charge-conjugation symmetry. Note that as far as I've been working here, the men correspond to positive particles and the women are negative. This is not symbolic.

So now we move on to the *orgy*. This is a rather disgusting topic so I won't spend much time on it. However, it relates quite closely to one of my favorite areas of physics, that is, nuclear physics. Those people who know me think I am more interested in nuclear physics than in girls; ask Melissa if you don't believe me. I mean, the nucleus is just plain cool. I know people who think it's cool to hold a burning cigarette in their hand. How could that ever compare to holding an acrylic rod with ^{137}Cs in your hand? Its emissions [i] don't smell bad, [ii] can go much faster (sometimes even as fast as the speed of light, *c*), [iii] can pass through your lungs without interacting, [iv] make the Geiger counter buzz, and [v] despite all the crap you've heard from the anti-nuke crowd, are much much healthier than cigarettes. But anyway, I'm getting sidetracked here. Back to the orgy.

A typical orgy consists of protons, who we call men, and neutrons, who we call women. The only stable orgies with more protons than neutrons are ^1H (1 man) and ^3He (2 men, 1 woman). Other than that, the following principles appear to hold:

Proton-Neutron Symmetry. An orgy typically has roughly equal numbers of men and women. In large orgies, there are more women than men because the men repel each other, that is, they all stink.

Parity. Even numbers of men and even numbers of women are preferred. The only odd-odd pairs which are stable are ^2H, ^6Li, ^{10}B, and ^{14}N. Of these, it is worth noting that the first two are unstable at high temperatures in stars, and the third (^{10}B) is very good at soaking up stray women, especially the ones who are thermalized.

Beta Decay. If there are too many women or too many men for the total number of men and women in the orgy, some will undergo sex-change operations performed by the weak force. This is accompanied by the emission of beta particles (e^- and/or e^+) and neutrinos.

To explain these and other properties, we apply a shell model to the orgy. The orbitals in which men and women can go are similar to those of atomic structure, except here we can fit two protons and two neutrons into each orbital. The ordering of energy levels is now: 1s, 1p, 1d, 2s, 1f, 2p, 1g, 2d, 3s, 1h, 2f, 3p, 1i, 2g, ... Note that the low energy of the 1s state predicts a stable orgy with two men and two women; this is known as the a-orgy. Also note that if there are many more men than women, the men's Fermi energy exceeds the women's Fermi energy, as a result there is a decay in which a man changes into a woman, allowing him (her?) to occupy a lower-energy state.

Finally, we note that orgies of more than 209 members are inherently unstable by comparison to nuclear physics. These decay either by emission of a-orgies or by spontaneous fission.

Exercises.

1. This problem has several parts. It uses the breakup of simple (one man, one women) romances to determine the fine structure constant $a_{love}=e^2$ for romantic structures.

(a) Guess the number of kilocalories "she" burns when she breaks up with "him" through crying, etc. Determine whether this is the dominant part of the ionization energy of the relationship by comparing it to the amount of crying, etc. "he" goes through.

(b) Estimate the reduced mass of "him" and "her". The reduced mass of two objects of mass m and M is defined as m=mM/(m+M).

(c) Using the result from the Bohr theory of the atom that the ionization energy E is given by $(^1/_2)a^2mc^2$, where c is the speed of light ($3x10^8$ m/s) and a is the fine structure constant, determine the fine structure constant for love.

(d) The fine structure constant for the electromagnetic force is 1/137. Since the fine structure constant is a measure of the strength of the force (zero fine structure constant = force isn't even there, ~ 1 implies the force is quite strong), you are now in a position to compare the strength of love to that of electricity and magnetism. Comment on the deep philosophical importance of this comparison.

2. Determine what element or ion you correspond to. Be honest.

3. Explain why in the orgy shell structure, the first p level is below the second s level in energy, whereas in atomic structure the first p level is above the second s level and in a perfect Coulomb potential they are at the same level.

4. Estimate the energy (in GeV) of a typical nuclear transition in an orgy. Do you think Mossbauer spectroscopy for orgies is feasible?

5. Define an isospin for orgies. What is the total isospin quantum number T for the ground state of an a-orgy? Of ^{3}He? What are the physical spins I?