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Fractals and heavy tails on the Romanian Stock Market

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1. Motivation

- Stock market is an environment dominated by uncertainty.

- Hurst exponent and tail index of a stable distribution could be used to characterize uncertainty.

- We study the link between these measures of uncertainty and the likelihood of extreme negative returns.

2.1. Literature

- **Wang et al.(2010):** Stock markets as complex systems- generalized entropy principle.
- **Peters (1994)** – Fractal Market Hypothesis.
- **Kwapień, J., & Drożdż, S. (2005) -** Multifractality in the stock market.

2.2. Hurst Exponent

Fractional Brownian motion:

- $E(p_t) = 0, \forall t$.
- $E(p_t p_s) = \frac{1}{2}(t^{2H} + s^{2H} - |t-s|^{2H})$, $\forall t, s$.

H is the Hurst coefficient, $H \in (0,1)$, which defines the behavior of the returns series.

- H=0.5 - prices follow a random walk process, and returns are not correlated;
- H<0.5 - the series of returns presents positive autocorrelation (persistent series);
- H>0.5 - the series of returns presents negative autocorrelation (anti-persistent series).

The fractional Brownian motion has the property of auto-similarity of a fractal, because in distribution terms, we have: $p_{at} \sim |a|^{2H} p_t$.

2.3. Stable distributions

Stable distributions have a remaquable property: they allow for skewness and heavy tails and more, any linear combination of stable independent variables is also stable. In other words, the shape of distribution is preserved under linear transformation.

In literature there are several parametrizations of stable distributions. We chosed for this paper the parametrisation S0, in Nolan (2001)'s variant.

Thus, a variable X follow a stable distribution $S(\alpha,\beta,\gamma,\delta;0)$ if its characteristic function has the form:

$$\phi(t) = \mathbf{E}[e^{itX}] = \begin{cases} \exp(-\gamma^{\alpha}|t|^{\alpha}[1 + i\beta\tan(\frac{\pi\alpha}{2})sign(t)(|\gamma t|^{1-\alpha} - 1)] + i\delta t, \alpha \neq 1 \\ \exp(-\gamma|t|[1 + i\beta t\frac{2}{\pi}sign(t)(\ln(\gamma|t|)] + i\delta t, \alpha = 1 \end{cases}.$$

In the above notation $\alpha \in (0.2]$ is the characteristic parameter (tail index), $\beta \in [-1,1]$ is the skewness parameter, $\gamma \in (0,\infty)$ is the scale parameter and $\delta \in \mathbf{R}$ is the location parameter.

2.4. Fractals, heavy tails and extreme returns

$$P(Y_t^* = 1) = \frac{\exp(\beta_0 + \beta_1 H_t + \beta_2 \alpha_t)}{1 + \exp(\beta_0 + \beta_1 H_t + \beta_2 \alpha_t)}$$

- $Y_t^* = 1$ for lower tail of returns' distribution(5%).

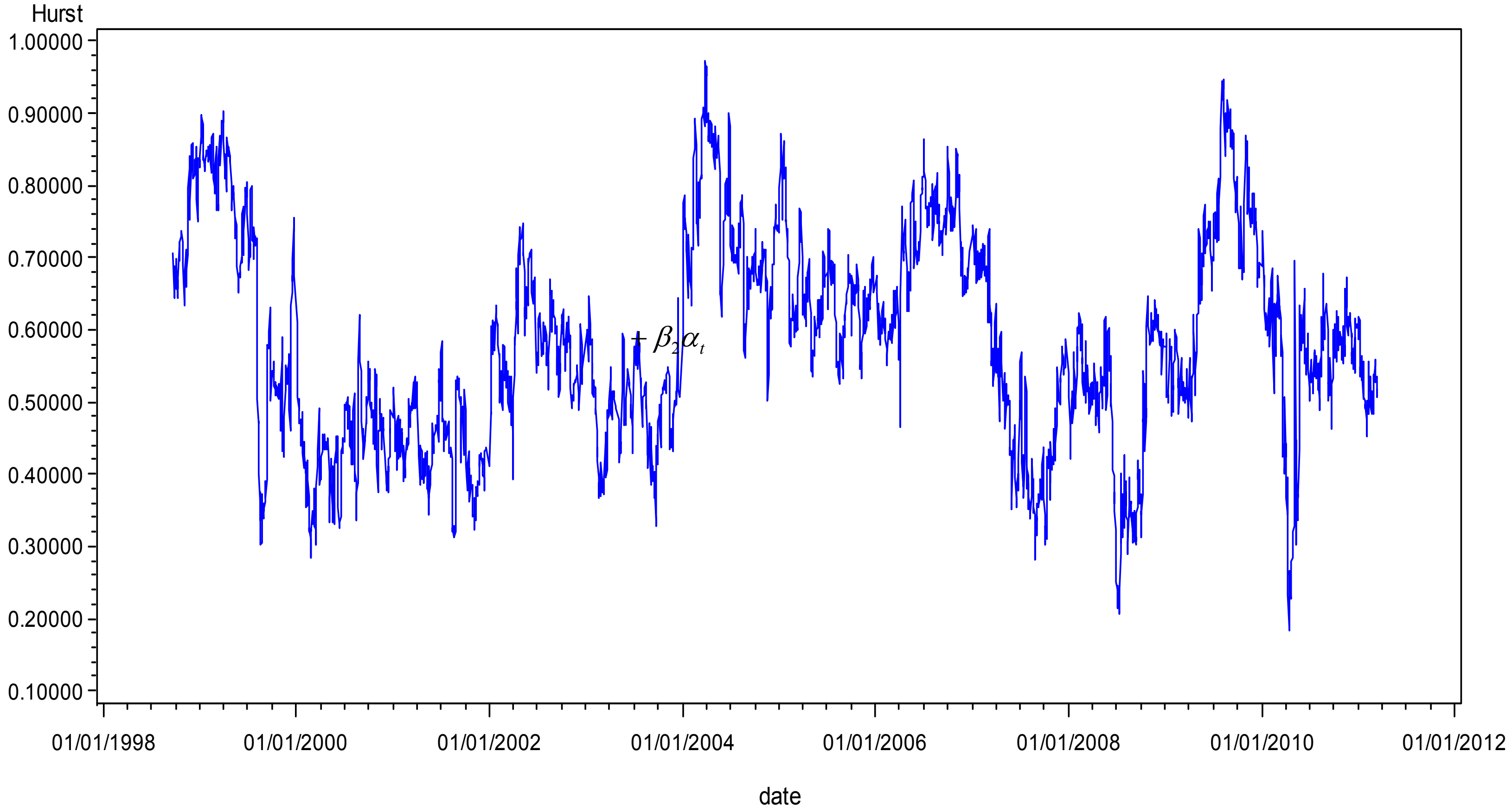
-H_t – Hurst exponent.

-α_t - tail index of a stable distribution.

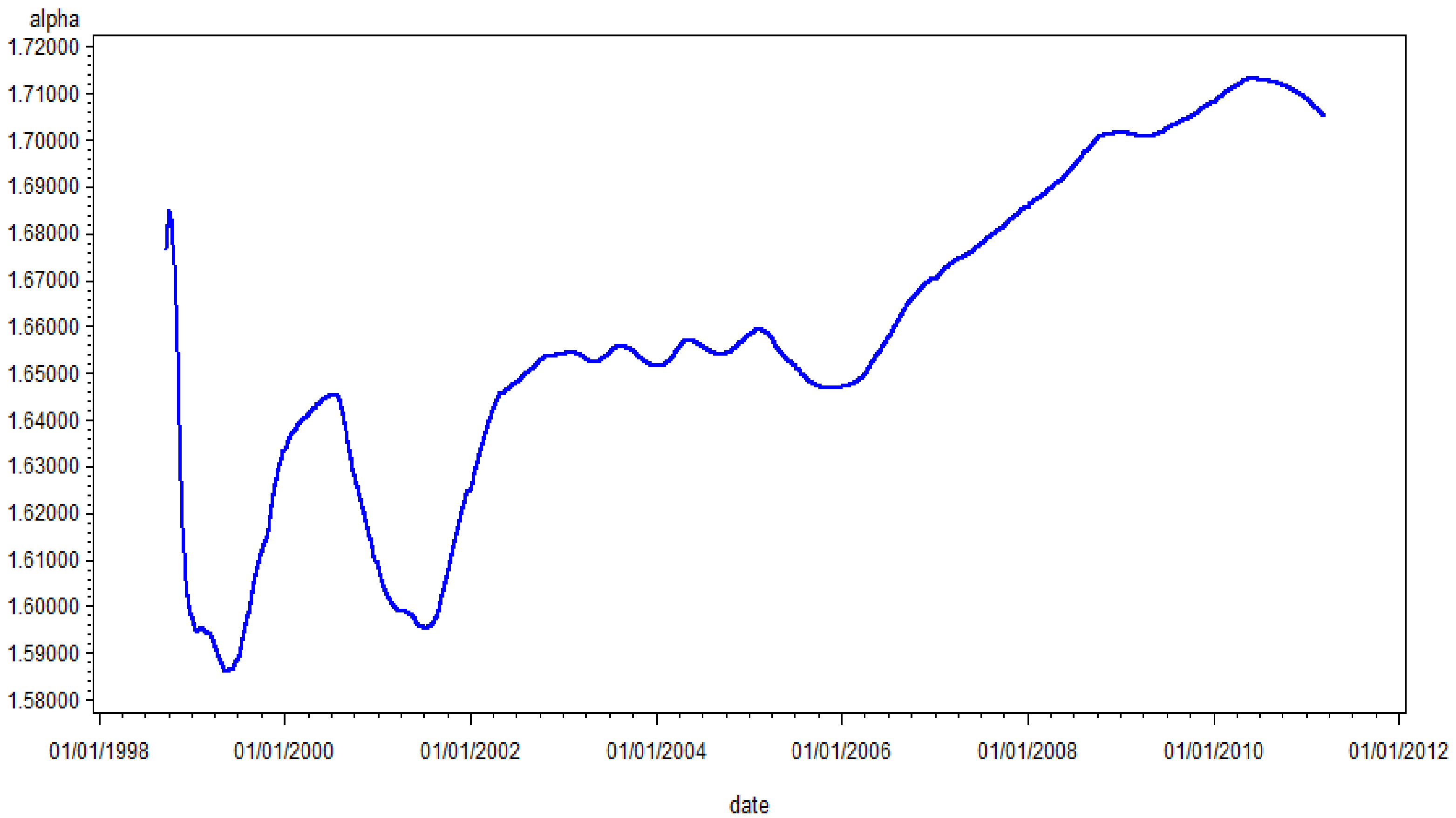
3. Results

- Daily data for BET Index (3357 obs. – 2007-2011).
- Rolling windows: 250 trading days.
- Hurst exponent - R/S analysis.
- Tail index – time series regression (Kutrouvelis method).

3.1. Hurst exponent for BET Index



3.2. Tail index for BET



3.3. Likelihood of extreme negative returns

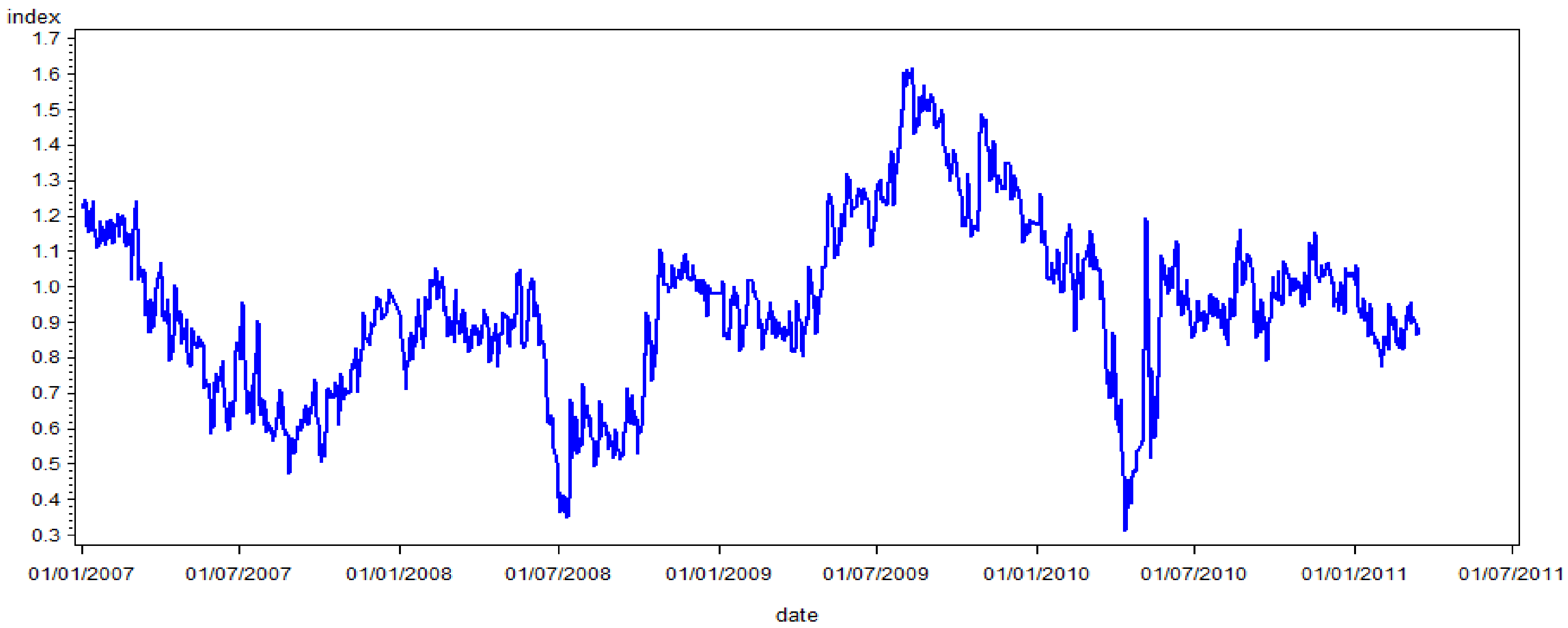
$$P(Y_t^* = 1) = \frac{\exp(\beta_0 + \beta_1 H_t)}{1 + \exp(\beta_0 + \beta_1 H_t)}$$

Analysis of Maximum Likelihood Estimates					
Parameter	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	1	-2.0900	0.3323	39.5476	<.0001
Hurst	1	-1.5095	0.5817	6.7336	0.0095

Odds Ratio Estimates			
Effect	Point Estimate	95% Wald Confidence Limits	
Hurst	0.221	0.071	0.691

3.4. Uncertainty Index

$$UI_t = H_t \alpha_t \in (0,2]$$



3.4. Uncertainty Index

$$P(Y_t^* = 1) = \frac{\exp(\beta_0 + \beta_1 UI_t)}{1 + \exp(\beta_0 + \beta_1 UI_t)}$$

Analysis of Maximum Likelihood Estimates					
Parameter	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	1	-2.2242	0.3322	44.8304	<.0001
index	1	-0.7646	0.3479	4.8292	0.0280

Odds Ratio Estimates			
Effect	Point Estimate	95% Wald Confidence Limits	
index	0.466	0.235	0.921

4. Conclusions

- Large negative returns are correlated to high values of Hurst exponent.

- It is possible to define an uncertainty index in order to explain large deviations in stock market returns.