

**A THERMAL MODEL OF THE ECONOMY**

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Abstract of Thesis Presented to the Graduate School  
of the University of Puerto Rico in Partial Fulfillment of the  
Requirements for the Degree of Master of Science

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Luis Balbino Arroyo Colón

The motivation for this work came from an interest in Economics (particularly since the 2008 economic downturn) and a desire to use the tools of physics in a field that has not been the subject of great exploration. We propose a model of economics in analogy to thermodynamics and introduce the concept of the Value Multiplier as a fundamental addition to any such model. Firstly, we attempt to make analogies between some economic concepts and fundamental concepts of thermal physics. Then we introduce the value multiplier and justify its existence in our system; the value multiplier allows us to account for some intangible, psychological elements of the value of goods and services. We finally bring all the elements together in a qualitative system. In particular, we attempt to make an analogy with the Keynesian Multiplier that justifies the usefulness of fiscal stimulus in severe economic downturns.

Resumen de Disertación Presentado a Escuela Graduada  
de la Universidad de Puerto Rico como requisito parcial de los  
Requerimientos para el grado de Maestría en Ciencias

## UN MODELO TÉRMICO DE LA ECONOMÍA

Por

Luis Balbino Arroyo Colón

La motivación de este trabajo surgió de un interés en la Economía (particularmente desde la crisis del 2008) y un deseo de utilizar las herramientas de la física en un campo poco explorado. Proponemos un modelo para la economía e introducimos el concepto del Multiplicador de Valor como una contribución fundamental a cualquier modelo similar. Primeramente, buscamos hacer una analogía entre algunos conceptos económicos y conceptos fundamentales de la física termal. Más aun, introducimos el Multiplicador de Valor y justificamos su existencia en nuestro sistema, ya que nos permite tomar en cuenta factores intangible, de naturaleza principalmente psicológica, del valor de los bienes y servicios en la economía. En particular, establecemos una conexión entre elementos de nuestro modelo y el Multiplicador Keynesiano que justifica el uso de estímulo fiscal en la economía en tiempos de crisis económica severa.

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# CHAPTER 1

## EXECUTIVE SUMMARY

We propose a relationship between certain physical concepts and basic economics, with a focus on Keynesian Macroeconomics. The statistical element present in such economic systems as the stock market suggests a connection with physical systems based on the statistical behavior of large groups of particles. We examine the example of the 'quants'<sup>1</sup>, who used methods familiar to the branch of Physics known as statistical mechanics to predict stock market fluctuations. Their predictions enjoyed a degree of success, but could not foresee the eventual crash of 2008. Our belief is that the psychological elements of the stock market, and the economy, are beyond the reach of a pure physics model. We propose a new concept, the value multiplier, to take into account factors that are beyond the scope of usual physics.

The work done by Georgescu and others is examined in the context of a direct equivalence between low entropy content of goods and high economic value. However, problems with that model force us to consider alternatives to that older system. We then examine macroeconomics and microeconomics, with a view towards proposing a relationship between microeconomics and statistical mechanics. That plausible link inspires us to propose a further link between their macroscopic counterparts, macroeconomics and thermodynamics. Because Keynesian (and eventually Endogenous Growth Theory) economics seems to us to be much more focused on

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<sup>1</sup> A certain group of particle physicists, unemployed thanks to federal budget cuts, went into finance to put their knowledge to use in financial markets.[1]



the intellectual and emotional elements that current thermoeconomic models ignore or underestimate, we choose that view of macroeconomics.

Our own model focuses on the importance of economic growth. We emphasize in particular growth that does not involve increased resource consumption. We build our model upon the classical Carnot heat engine, the usual template for their physical behavior. We choose heat engines because they represent the conversion of raw heat energy into useful work. A useful analogy is burning fuel, which accomplishes very little in comparison to burning fuel in a heat engine, which is the heart of our modern world. The heat engine model, however, just turns resources into goods without truly focusing on other things. That reason, therefore, is the logic behind the concept of the Value Multiplier.

The Value Multiplier is the way our model treats the growth in the economy that does not come from increased resource usage. It responds to a vision of price that is market determined. The profit margin, or the ratio between price and cost, is equivalent to the Value Multiplier. The sum of all those accumulated Values added to the resources available is the main source in our model of economic growth. Endogenous Growth Theory predicts that those Value Multipliers will come from increased knowledge about the use of resources that makes more efficient and widespread consumer consumption. Our model also considers the accumulated Value Multiplier that occurs in the pricing of derivatives in securities markets. Derivatives are basically methods to find that the Value Multiplier of a group of products is greater than the sum of its parts.

Our model also attempts to predict the Spending, or Keynesian, Multiplier. It uses the thermodynamics of the basic heat engine model to predict that government spending could boost the economy in temporary fashion. It also separates the Value Multiplier, a fundamental source of growth from the different kind of growth that comes from stimulating the economy. However, we do not oppose stimulus spending

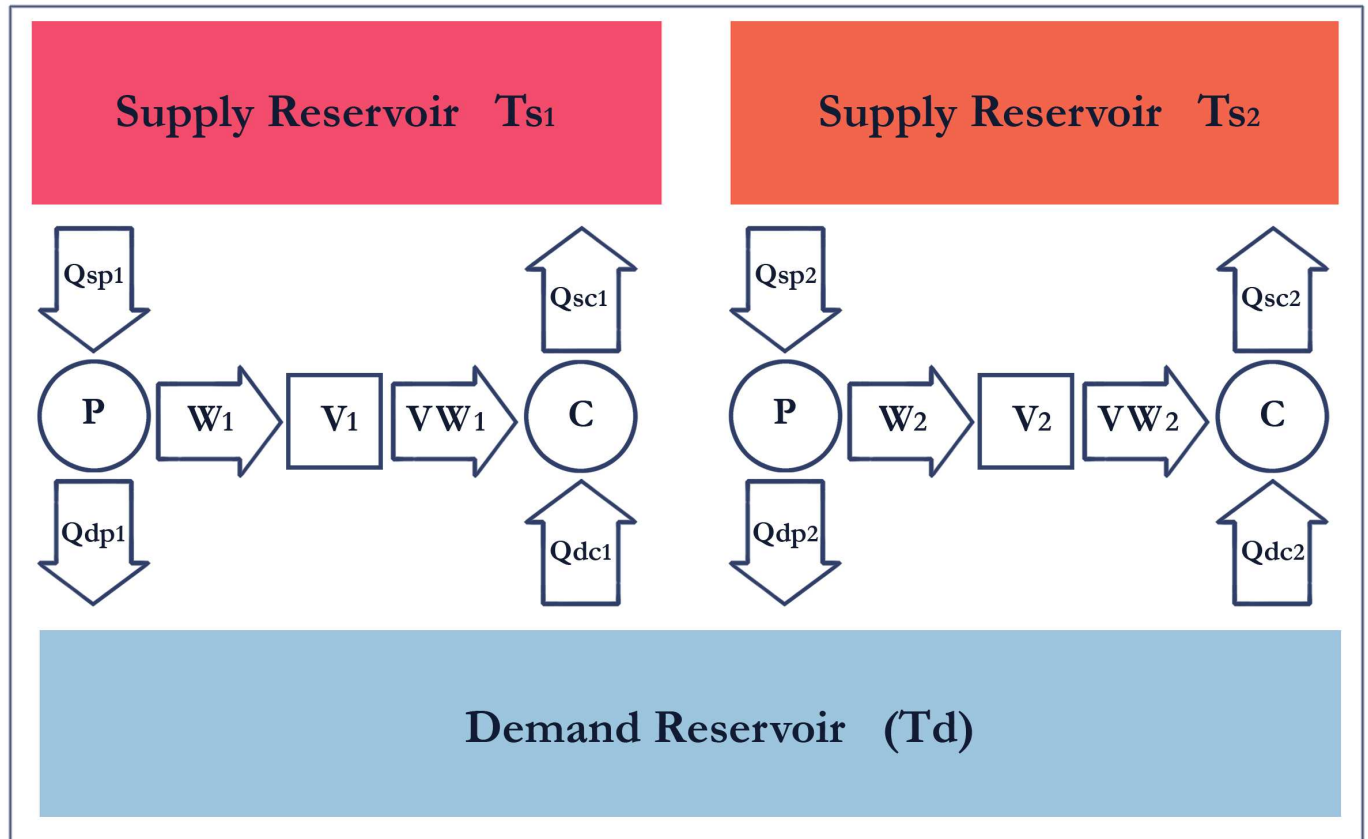


Figure 1-1: This is a Sketch of the Heat Engine System.

as a necessary part of government stewardship of the economy. The model only considers the first order effects; there is the real possibility that the psychological effects of a prolonged crisis would diminish the long term potential of the economy. We simply separate both concepts, as the model does. In conclusion, our model attempts to make an analogy between physical theory and economic concepts.

## CHAPTER 2

# INTRODUCTION

Why mix physics and economics? Although both are sciences, they are of a different nature. Physics deals with the reality of matter and its movement. Economics is mostly preoccupied with the production and consumption of goods and services. Those two sciences are, at first glance, totally unrelated. However, economics has a fundamental disconnect inside it that resembles some of the dichotomies present in a branch of physics.

Economics has two main branches: macroeconomics and microeconomics. Microeconomics deals with economic behavior on the small scale; with individuals and firms and their interaction. Since we assume market economics, we only consider markets in limited form. However, macroeconomics is not that simple, because it deals with the aggregation of economic systems.

Aggregation changes everything; the complexity of it all increases dramatically. That complexity makes it more than the sum of its markets. The nature of macroeconomics cannot be explained by simply adding up the micromarkets. Some of the biggest controversies in macroeconomics at this moment involve the need or lack of a microeconomic foundation for Keynesian economics. In particular, the role of inflation and the money supply in overall GDP<sup>1</sup>

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<sup>1</sup> The GDP is the Gross Domestic Product. There is more than one way to calculate it. However, for our purposes, it is the sum of all outputs has been a source of discussion. Microeconomics would tend to believe that the only thing that could affect production levels is relative pricing, not the overall price level, related to

The situation in micro and macro parallels that of thermodynamics in the 19th century. Back then, physics had found the general laws of thermodynamics that ruled the behavior of heat and work in thermal systems. However, the atom had yet to be discovered, and the true laws of microscopic behavior of particles were far off. Empirical observation led to the formulation of thermal laws, but the true cause of them was out of reach. Only with the rise of quantum mechanics in the beginning of the 20th century could physics truly explain the behavior of the individual atoms that when united become thermal bodies.

Indeed, the quantum behavior of atomic bodies is so greatly different from the phenomena we see in everyday life. We would dare say the key difference lies in the Heisenberg Uncertainty Principle. That principle clearly states that what we can know about a body is limited by the laws of physics. We cannot know precisely where a particle is at the same time we know precisely where it will be in the future. That limit is not epistemological, but ontological. Nature itself acts as if it did not know, because it cannot know. Put in a different way, the amount of information that we can have about anything is limited by physics; a limit that does not come from our limitations in measurement, but is inherent to nature.

The limit on information gives rise to the concept of entropy. Since we cannot totally know the state of the system up to the smallest level, we measure the disorder of the systems by counting the different ways the system can be ordered in the small scale consistent with the macroscopic state of the body we observe. The logarithm of that count, multiplied by a fundamental constant of physics (the Boltzmann

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the money supply. The Keynesian model, which we will follow in this work, cannot be squared immediately with the microeconomic theory we understand today. [2]

constant), becomes the entropy:

$$S = k_B \ln W. \quad (2.1)$$

And, in a curious parallel, one of the key concepts in economics is information. In particular, how information imbalances in a market can lead to inefficiency or even failure. And that could be a key bridge between the micro foundations that macro must have, and the empirical observations that Keynesian economics has been based upon.

The micro and macroeconomic perspectives cannot be easily reconciled. We know too little of human behavior, a key factor in establishing the transition point where micro becomes macro, where the sum of all the activities behaves different from its component parts. One of the hopes we have for this work is the illumination by means of analogy of that boundary. In physics the boundary between the micro and the macro has been explored with more success. Even though we deal not with objects but with people, the similarities we have discussed earlier may give us a hint of micro-macro transition points. That could lead us to a better understanding of the boundaries between them.

Maybe that difference is related to the possibility of a limited bandwidth, or capacity for considering multiple data points at the same time, of the human mind. That uncertainty could lead to emotional responses that are unpredictable from a rational standpoint. And that lack of information could result in a difference between the numbers we see in the real economy and the numbers we should see if people were rational. And that uncertainty could be described by something akin to the Uncertainty Principle, the fundamental difference between the micro and the macro in physics.

To talk about the analogy between economic and thermal systems should not be misinterpreted as meaning that this model is a perfect analogy. The number of

concepts involved is too large and the intricacies of physics too complex for a simple and complete equivalency. Physics is a natural science, and it is both the most mathematical and the most exact. Economics, however successful, is at heart a social science. Human behavior is ill-suited to exactness and mathematical certainty by its very nature. We should not confuse the presence of many formulas and numbers as meaning that this theory is a total model, or that it pretends to be perfectly predictive. This work only seeks to establish fundamental concepts, and cast new light on familiar ideas from both economics and physics. In such complex and, at first glance, dissimilar sciences, this work seeks to be humble in its ends.

## CHAPTER 3

# PREVIOUS WORK

This work is focused on using physical models and concepts to model economic systems. We propose the use of statistical mechanics and thermodynamics to further our understanding of macroeconomic phenomena. We lay out the motivation and the mechanism of our economic 'machine.' To better understand what we propose to do here, we must examine the context of economic modeling using physics in the scientific literature.

There is a healthy body of work in economic and physical models. One such model, and quite possibly the most referenced, is the one in the book 'The Entropy Law and the Economic Process' by Nicholas Georgescu. It is fundamentally predicated on the fact that all economic goods must be physical goods. The main proposition is that a production mechanism takes resources (mostly land, capital and laborers) and creates goods. In his view, the main economic innovation is the factory, which attempts to make ever more efficient the usage of limited resources.

In his view, resources are reduced to their energetic content. Obviously, energy is a critical economic resource, and it also is a physical reality. Of course, this means that economic growth is limited, because the total energy is conserved. He considers a production process as merely the transformation of energy (including energy in the form of physical resources). Indeed he views a factory as a big heat engine. It turns raw resources into work. As we know, the difference between equal amounts of heat energy and work (which is a form of energy) is the change in the entropy of the bodies caused by the energy flow. And to reduce the entropy of an amount

of energy takes additional energy that is not added to the work. That additional energy is the price of reducing the entropy.

He differentiates inputs and outputs by their entropy change. We know that raw materials (physical materials) become finished products. Part of the raw materials are lost as part of the production process (such as burnt oil to power the factory). Of course in production systems, the human beings do not become part of the production either (we assume the sausage factory does not malfunction). The rest of those resources are the products, specifically that part of all the inputs that went into the factory that saw its entropy go down. Of course, we must emphasize that entropy can only stay the same or increase in global terms. However, part of the resources can have lower entropy, compensated by the increase of entropy elsewhere.

This correlation of turning resources into products and entropy reduction is the heart of the model in [3]. This allows us to clarify the relationship between the economic point of view and the physical perspective in the model proposed there. Economically, a factory exists to produce higher value material than what came in. Physically, it is impossible to create energy, but we can decrease the entropy of part of the material. Therefore, Georgescu makes the connection between value and entropy. The lower the entropy, the higher the value. The exact relationship is not established, since the author realizes that to insist on a strict and unbending relationship would be folly. However, he does believe it is a monotonic relationship.

Our own model shares the focus on energy and entropy. It could hardly be different when we are discussing statistical mechanics and thermodynamics. However, it is in the shape of the analogy between the economics and the physics that our model is different. Firstly, the whole book deals with factories, old-fashioned industry, and production. The service industry is not considered, because it deals mostly with knowledge and labor, not with physical resources being consumed.



Second, consumption is not given much consideration. This is a natural consequence of the model. Production is the critical apart of the process, since all that is produced is consumed by definition.<sup>1</sup> The more resources consumed, and the higher thermodynamic efficiency attained, the better. And of course, when low entropy is the whole source of value, how could the consumer matter? The consumption of the good is predetermined by the entropy consumed. This would be in direct contradiction to empirical evidence. The whole marketing industry would be redundant, because marketing would not matter. This leads to a more fundamental problem with the model proposed in [3].

The model does not take into account the creation of wealth absent increased resource usage. For example, technology has a value that cannot be determined by energetic considerations. Indeed, if we were to enforce an impossibly strict energy accounting system and calculate the whole amount energy consumed by a given technology and the whole amount energy saved or unleashed by the same technology, they would differ. A purely thermodynamic model is insufficient. A good example is land. The deserts of Arabia were relatively worthless a few centuries ago, yet the crude oil beneath them that was discovered relatively recently and the creation of an oil industry has increased dramatically the value of that land. This cannot be explained by pure physical reality, because the lands have been basically the same for eons.

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<sup>1</sup> This very idea is fundamentally similar to the rational expectations models of so called 'freshwater economics'[4]. It stands dissimilar to Keynesian macroeconomics. The very idea that all that is produced is consumed, and the focus on the supply side to the utter exclusion of demand, is essential to the freshwater school of economics Krugman refers to. This should not surprise, for one of Georgescu's critical influences is Schumpeter, one of the historical sources for the freshwater school and a physicist himself.

Furthermore, even if we assume that the 'true' value of something can be determined by purely thermodynamic considerations, that is not the way reality works. People buy based on their preferences. Those preferences are based on the economic concept of utility, which is maximized. Economic growth is also related to specialization of labor. The concept of comparative advantage is essential to economic theory. It increases the total production of the economy, making everyone better off. A true economic model based on physics should be more comprehensive in the phenomena it explains.

A possible explanation for the high emphasis on production in Georgescu's work is a sign of the times. In the early 20th century, economies were either agricultural or industrial. They involved the use of energy, mostly solar in agriculture and fossil fuels in manufacturing, to produce goods. However, in our times, the service sector has become the true center of growth for the world economy. As its very name points out, services do not produce physical goods that have entropy. Services performed reduce entropy on other bodies, and they have an energy cost, much like the production of physical goods. However, to quantify that entropy reduction in services is much more difficult than the more purely physical entropy reduction on the inputs of a production process. When a factory produces goods, it is (in theory, ignoring the obvious complications) straightforward to do a strict accounting of the energy that came in, and the energy that came out. However, when a service is performed on a human being, the question of calculating the entropy change in the human is an imponderable. Since services weren't an important part of the economy in the 1930's when many of the ideas he discussed were being developed, it is no wonder that Georgescu did not consider them in his theories with any degree of importance. And when the book that we cite came out, in the 1970's, science discovering that limits on natural resources were approaching; the materialistic approach he took must be placed in its context.

### 3.1 Review Article on Econophysics

Jurgen Mimkes wrote a review article on Econophysics [5] that goes over most of the econophysics models in current usage. It mostly references other articles to create a somewhat comprehensive picture of the latest developments in the field. In fact, most of the concepts that have been proposed seem to be derived from [3]. The only differences reside in the article, the author specifies the use of Carnot cycles in the heat engines, and mentions consumption. However, consumption is not mentioned as an important activity, but as merely a cog in the production process, whose results are inevitable.

The same as in Georgescu's work, Mimkes tries to relate directly money flows to energy flow. Indeed, since lowering entropy is the objective of production in the model in [3], they relate the production function to entropy. Now is a useful moment to discuss what the production function is. Basically, it represents the value of a good that can be produced given a certain amount of capital and labor[6]. Therefore, it is possible to compare both concepts and establish an analogy, if we assume entropy diminution is at the center of the model.

However, the same fundamental problems that apply to Georgescu's work are present in [5]. However, their contribution is important in establishing that certain analogies can be made between economics and physics, specifically thermodynamics.

### 3.2 Thermodynamics in the Economic Mainstream

Much closer to the usual economic thought, much of economics operates on two fundamental assumptions: maximization (of profits for firms and utility for consumers) within budget constraints. In his Nobel Prize lecture, Dr Paul Samuelson used thermodynamical concepts to illustrate the thought behind the twin pillars of equilibrium analysis: maximization (in microeconomics, of utility) within constraints (the budget constraint on spending) that produces stable equilibrium[7]. There is no justification for the assumption of maximization beyond the obvious empirical

observation that people tend to maximize their own welfare. More problematic is the assumption of stable equilibrium that permeates the system. Although the stable equilibrium is a valid assumption in stable times, it loses usefulness in dynamic times.

Equilibrium of a function of quantity  $Y$  occurs when its derivative equals zero, such that (if  $T$  is the time)

$$\frac{dY}{dT} = 0. \quad (3.1)$$

The stability of the equilibrium does not depend on the derivative being zero; it depends on the second derivative. If it is positive, the equilibrium will be unstable, as the system will reinforce the perturbation introduced. If it is negative, then the equilibrium will indeed be stable. In addition there may be many possible equilibriums.

In order for that concept to be applicable in all situations, and totally predictive of behavior, human beings must be rational machines whose behavior is totally predicted by rational circumstances. That reason must not be swayed by emotions easily, since the forces that sustain the equilibrium collapse. Reason lends itself to simple models of convenience and obvious causality. A useful, and well worn, analogy is a box hanging on a spring. In general, as is well known to freshmen physics students, that equilibrium is stable. If the box be initially stationary (at equilibrium, since it feels no net force), and then a perturbation is added (moving the box down a few inches), the box will continue oscillating around equilibrium until friction eventually strips its extra energy and it returns to equilibrium. However, if the jolt is strong enough, the spring may deform or even break, and the equilibrium that existed will change; equilibrium could well become impossible. Big jolts make the concept of equilibrium in that seemingly simple and predictable system untenable.

Although stable equilibrium is important for most things in economics, financial systems should not lend themselves well to that dynamic. Reason has little place

there, as bulls and bears battle it out in the exchanges. The crowd mentality lends itself poorly to logical analysis and much less to stability. In times of stress, both in the upturn and the downswing, economics must have its greatest usefulness, yet the persistent use of equilibrium techniques may harm more than it helps. Big jolts may destroy the foundation of the stable equilibrium formulation, without talking about multiple possible equilibriums.

The equilibrium model, as Samuelson himself points out, is less useful when dealing with downturns in dynamic situations. Our model focuses particularly on that part of economics when we will use the Value Multiplier to illustrate a part of the Fiscal Stimulus Multiplier. In particular, a critical finding and summation of our concept of the Value Multiplier is

$$V \propto \frac{\text{Utility of Output}}{\text{Utility of Input}}. \quad (3.2)$$

That formulation is important, because utility is a human calculation. In times of economic uncertainty, utility (a quantitative measurement of human/psychological realities that lend themselves poorly to numbers) becomes unstable. Indeed, that change negates the fundamental assumption of stability of equilibrium. The introduction to the model of the Value Multiplier will serve to highlight that essential reality. This will be particularly true of economic flux caused by financial markets.

### 3.3 'Quants'

In recent days Wall Street has suffered the effects of the financial meltdown. Some of the blame for this has been deposited on physicists who were involved in the creation of some of the models that were found to be faulty during the crisis. [1] These models were taken in many ways from statistical mechanics. If we remember that statistical mechanics is basically the micro-scale foundation of thermodynamics, it is quite sensible to see that if some thermodynamic work could help understand economics, perhaps such statistical models could be of some use. For example, the

Black-Scholes equation, essential in the modeling of options<sup>2</sup> prices, is based in part on assumptions on the behavior of particles that are related to the Maxwell-Boltzmann probability distribution [8].

Although those physical models have proven controversial, they have had some use in the financial community. But those models have not been presented as deriving necessarily from a thermodynamical foundation, as statistical mechanics does. If a macro-scale study of thermodynamics and its relationship could be solidified, perhaps the use of statistical methods in financial markets could be better understood and applied.

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<sup>2</sup> Options are part of what has been called the derivatives market, the center of the current financial collapse. Derivatives are fundamentally securities whose value is based on the value of other underlying securities such as bonds, stocks, mortgages, among others.

## CHAPTER 4

# FUNDAMENTAL CONCEPTS

There are similarities between economic and thermodynamical concepts. For example, money is conserved in exchanges since it is not created or destroyed. Although money is not a resource in itself, it represents them and quantifies their relative worths as determined by the market and labeled by prices<sup>1</sup>. Since in a general sense we assume that the value of a good is its price, then value is conserved in any exchange of money for goods.

Another key similarity between thermodynamics and economics is the sheer complexity of the systems they study. Of course, we do not pretend to say that economic and thermal systems are similarly complex, merely that the jump from micro to macro in both increases exponentially the interaction and complexity. To describe a thermal system exclusively in microscopic terms would be impossible in practice because of its computational difficulty. Similarly, macroeconomic systems are described through statistical models and theoretical assumptions as to the behavior of the micro-components.

Economics is fundamentally related to human behavior. In many ways, it seeks to quantify the effect of consumption and investment on the standard of living. To increase the standard of living, ways must be found to harness the energy around us into useful forms. And in order to harness it, the amount of entropy must be

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<sup>1</sup> We assume throughout market determination of prices.

reduced. In other words, the only way we can do useful things with our energy is by turning the economic equivalent of heat into work.

#### 4.1 Macroeconomics and Microeconomics

What is microeconomics? Microeconomics is concerned with the behavior of individuals, families and small groups. It mostly examines markets, and how prices are set. Of course, the main working assumption of microeconomics is the settlement of prices by the interplay of supply and demand. The behavior of consumers and producers in reaction to changes in prices is important as well [6].

Macroeconomics deals with the behavior of aggregate markets. Aggregate markets are the sum of all the small markets that microeconomics takes into account. A single market can be studied by microeconomics, and subjected to sufficient study by itself. However, when dealing with the thousands of different markets in a real-world economy, it turns out to be practically impossible to study them one by one. Not only is each market different, the interactions, or 'cross-effects', between them can alter totally their behavior. To study macroeconomic systems, one has to analyze the behavior of aggregate measures, such as Gross Domestic Product or Inflation[9].

The contrast in scope between these two perspectives on economic behavior is reminiscent of the relationship between thermodynamics and statistical mechanics. The practical effect that both theories have is the same, since they both predict the same kinds of behavior. The ideal gas law, for example, is critical in thermodynamics. Yet it is a law that can be derived from first principles as a consequence of thermodynamics. As it should, because both offer different ways of looking at the same physical problems.

The same should be true for microeconomics and macroeconomics. Although both offer different perspectives on economic systems, the results they give should be similar. At the very least, then, there should be some attempt to reconcile microeconomic principles with what should be the working assumptions of macroeconomics.



The economist Robert Lucas has been a key proponent of relating both areas of economics. However, it has not proven easy. This is true because, in contrast to most computations in statistical mechanics, there are interactions between particles (or markets). Such interactions between markets, or cross effects, complicate things dramatically. In physics one can simply assume that there are no interactions and the results follow relatively simply. However, human beings are more than a little bit more complicated than that.

At the same time, if one embarked on a comprehensive study of the major component parts of a macroeconomic system, it would prove difficult if not impossible. Both frames have their importance and their use, and they should predict the same things. So in that sense the analogy between micro and macro perspectives is preserved. That is true in economics by definition, and the same can be said about thermal physics.

## 4.2 Economic Growth

We know that economic growth in its many forms is at the center of what economics desires. The increase in living standards that accompany economic growth has been demonstrated many times. Healthy growth done in a smart way increases the wealth of society. We know that Puerto Rico has seen both terrific stagnation and truly stunning growth. In a way, the sudden transition that occurred between solid growth and relative stagnation around 1974 has changed the way we look at our economy.<sup>[10]</sup> <sup>[11]</sup> The limited success and even abject failure of subsequent administrations at improving growth have taught us the difficulty of changing economic policies and the challenge of determining objectively what's best. So, in a way Puerto Rico's experience illuminates the importance of finding what creates growth and what does not.

First of all, we must answer the question of what defines growth. Most of the time it is defined as the increase in Gross Domestic Product. The GDP measures the

total value of what the economy produces in a given period. The value of the goods and services produced is the product of the amount of a given a good and its price. Of course there are different ways to measure the size of the economy; one example is GDP per capita which takes into account the fact that a large population can easily mean that a large GDP does not necessarily result in good living standards. The exact definition in our model is not important. The key factor here is that economic growth occurs in economic terms.

We mentioned earlier that GDP is related to the product of the amount of a good or service actually produced, and its price. The first quantity is relatively easy to understand. However, the setting of prices is of more concern to us right now. Human beings quantify value by setting prices by means of a market through the interaction of supply and demand. Therefore, markets can be said to set prices.<sup>2</sup>

To talk about the economy we have to see it as a cycle. Money is spent many times, and it is used both as payment for goods and as compensation for services rendered. That cycle both feeds on and is fed by the interaction between consumption, production and investment. Therefore, economic growth results from that complex interplay. But in order to create sustainable economic growth, based not just on increased usage of natural resources but on truly greater wealth, there must be innovation. If we wish to continue growing without eventually overstressing the resources of our planet we must rely further upon innovation, new ideas and methods. This insight will serve us well as we proceed to consider the economic model itself.

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<sup>2</sup> Although this will be discussed later, this strongly hints at the concept of Value Multiplier, or non-absolute conservation of energy. Since energy is closely related to value, if total value tends to change then the total amount of energy should change.

### 4.3 Production and Consumption in the Model

In economics, we can identify both short term and long term economic behavior. The long-term behavior is the trend of the economy's growth. If we subtract from the total change the trend, we will find substantial deviations around that trend. So, it is possible to separate trends from fluctuations. We as human beings are more sensitive to the short term behavior because for years, growth can be below trend, hurting the overall health of the economy and causing hardship for thousands of families. [11].

It is important to quickly survey the model we are proposing. At this stage, we simply have two heat reservoirs and two thermal cycles. One of the reservoirs is the hot (or supply) reservoir with a temperature  $T_s$ . The other (demand) reservoir is colder and has a temperature of  $T_d$ . As we discuss further in the appendix, a heat engine is basically a way of using heat to do work. In our model that process represents the act of turning resources into products and services, or production. Then that work is multiplied by what we call the Value Multiplier, or  $V$  which we will also discuss. Then that work is used to run a reverse heat engine, or refrigerator. That represents the flow of money as it goes back to the supply in the form of consumption. In other words, if we ignored the Value Multiplier, that system would represent an endless cycle of production and consumption. In many ways that system would run by itself in an endless natural progression.<sup>3</sup>

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<sup>3</sup> We must not confuse this statement with proposing that our system is a perpetual motion machine, prohibited by the laws of thermodynamics. Instead, we are saying that, since production and consumption are compared in our proposal with a refrigerator and a heat engine, and since we assume reversibility, what will happen is that every cycle the heat engine goes through will be reversed (effectively undone) by the other.

### 4.3.1 Money and Energy

When we think of economics, a word that could come to mind is money. However, money is not a resource in itself. A dollar bill has very little use in and of itself. Money is useful as a representation of the value of actual resources. The way we value things is relative to other goods or services. Therefore, money is the way the market quantifies value when it is at equilibrium[6].<sup>4</sup>

## 4.4 Heat Engines

We start with the concept of the heat engine. The heat engine fundamentally represents the physical mechanism through which heat energy becomes work. Although we discuss this topic further in the appendix, suffice it to say that heat engines are one of the most important concepts in thermodynamics. This is similar to the economic concept of using raw materials and labor to produce. Therefore we endeavor to make an analogy between them.

As is noted further in the appendix, both heat energy and work are forms of energy. The difference between them is the change in entropy induced by the flow<sup>5</sup>. Energy is removed from a hot reservoir that has a relatively high concentration of heat energy and passes through a heat engine to do work. To reduce the entropy content of heat energy enough, other energy must be expended. But since energy is conserved, the portion that did not become work is deposited somewhere. That somewhere is the cold reservoir.

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<sup>4</sup> Since all markets will hardly be at equilibrium constantly, there is a difference between market value and market price. Market price is what people will pay, which can be different to what the good or service is actually worth. However, since in thermodynamics we assume equilibrium, we will take both price and value to be the same. In other words, for our limited purposes here, we assume a rather permanent stable equilibrium so that market price accurately reflects actual trend or long-term value [6]

<sup>5</sup> The entropy content of a body will increase when heat arrives into it

The use of work in our model as a representation of production is critical. Heat energy is transferred solely to achieve thermal equilibrium between a few bodies. So, if there is a difference in temperature, heat will flow from the hotter to the cooler body. That is the gist of the Second Law of Thermodynamics. However, production does not behave like that. There is no natural process of acquiring wealth; it doesn't rub off. There are simply interactions based on decisions made to produce and consume, and the market price set for the particular interaction. The only way that behavior can be controlled is if that energy (or value) flow of goods and money can be directed, something that is impossible to do via heat transfer alone. Heat transfer is too random. Furthermore, since that heat energy flow is not work, it is accompanied by a transfer in energy.

If we have defined work as resources that have been harnessed with a purpose in mind, then we can also define heat. By definition, particles that have thermal or heat energy have no preferential direction of movement; they're all spinning around. If we take that as a hint, we can define heat energy as unharnessed or unused resources, waiting to be harnessed (energy in the hotter reservoir, waiting to become work) or 'wasted' as part of the production process (the heat energy left over from the heat engine)<sup>6</sup>.

#### **4.5 The Second Law of Thermodynamics and Production.**

We have mentioned that Thermodynamics prohibits the total conversion of heat energy into work. This is a more stringent requirement than mere conservation of energy (The First Law of Thermodynamics) because it limits the efficiency of a heat engine. This law places an inherent limit on the efficiency of production.

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<sup>6</sup> As we will see later on, this energy is not wasted, it is simply what feeds the demand in a way. It represents largely labor costs, and that labor will consume the goods and be useful for production again in the hot reservoir.

If we extrapolate the concept of limited efficiency to economics, we can propose a strong analogy. Production processes have inherent inefficiencies. Not of all the resources that are consumed in a factory end up **as part of the product**. For example, the calories needed for a human being to survive and work are not trivial. Indeed, it was the experience of Nazi Germany that dramatically cutting food consumption for workers was a disaster. [12]. Therefore, we have no choice but to conclude that even the most perfect production processes has an upper limit on its efficiency, even assuming that all goes according to plan.

An upper limit on efficiency then solidifies our case for using thermodynamics as part of the economic model. Indeed, this use of the Second Law of Thermodynamics has deeper meaning. Fundamentally, the reason why there is a limit on theoretical efficiency in physical heat engines is the statistical behavior of large numbers of particles. Translated into economic terms, that means that the reason why there is an upper limit on production processes would be statistical: the more complex<sup>7</sup> a production system is, the lower its efficiency will tend to be. This is due to the greater number of possible microstates; the more microstates consistent with the macrostate, the more entropy.

#### 4.5.1 On Temperature and Economics

Thermodynamically, temperature is the quantity that decides which way heat flows. In mathematical terms,

$$\frac{1}{T} = \frac{dS}{dQ} \quad (4.1)$$

where  $S$  is the entropy and  $Q$  is heat energy (not any kind of energy, only heat energy). Therefore, temperature is the ratio that determines the entropy change

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<sup>7</sup> Entropy (or disorder) is defined in terms of the amount of different possible configurations a system has. The more 'moving parts' a factory's, or its analogue's, resources has, the higher its entropy will be. Higher the entropy means less efficiency.

given flowing heat energy. It is important to note that this is only true of reversible systems<sup>8</sup> ; in other circumstances the equality breaks down. For our purposes, we assume reversibility throughout.

We have established that we will use the real value of resources, as determined by the market, as a proxy for energy. We have mentioned entropy and established a degree of analogy with economic concepts. How will we define temperature? In physics, temperature tends to determine average energy per particle; high temperature results in higher energy per particle. In a similar vein, temperature would be a kind of energy density. This would point to temperature being a measure of the density of resources available to a reservoir, or group of particles which will do work through a heat engine.

We can refer to a group of people as a reservoir, since people are or own resources. The higher the wealth of the people in the reservoir, the higher the average temperature will be.

#### 4.6 In Review

So, in the same way that heat engines are the mechanisms that thermodynamics use to turn randomly distributed and stored energy into actual work; the basic concept of production is to organize capital, labor and land into goods that can be both saved and consumed. From scattered resources (including money) that are not directed towards any direction or specific purpose, something useful is created. However, to create something useful out of randomly directed (or undirected) resources, some of them must be expended to organize the remaining resources.

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<sup>8</sup> Reversible systems are those that can be run in reverse and then return to the original condition.

# CHAPTER 5

## PRODUCTION, CONSUMPTION, AND HEAT ENGINES

In order to have a Heat Engine to model production, we have to define the elements of the heat engine. We need two reservoirs with different temperatures and a thermodynamics cycle that describes the method in which heat energy becomes work. First, we must define the reservoirs in economic terms.

There must be a temperature difference between reservoirs, and temperature has been defined as resource density. We can define the hotter reservoir as a supply reservoir. Such a supply reservoir is where resources lay waiting to be tapped and used. We name it thus because the more resources are available there, the more that can be potentially produced all else being equal.

### 5.1 The Reservoirs

The supply reservoir contains energy, or in economic terms, resources. Those resources can be used, and the amount of resources used is taken from the hot reservoir. We call the amount of heat energy the supply reservoir gives to the heat engine  $Q_{sp}$ . The notation we use here is this:  $s$  represents energy entering or leaving the supply reservoir;  $p$  represents the fact that this energy is part of the production process.

The second reservoir has to be colder relative to the supply reservoir; otherwise the heat engine would not work. However we assumed that particles are people that own resources, such as land, capital and their own labor. So when someone is 'in' that reservoir he has expended his resources. We have mentioned that the economy



has a cyclical quality to it. In a sense, after people work they consume, and pay for their consumption with their wages and the rents on their resources. The particles still retain a value,  $Q_{dp}$  where the  $d$  represents the second or Demand reservoir.

It turns out later that there is an amount of energy that leaves the demand reservoir as part of consumption; those are the resources that were not tapped, but that can be tapped later. In our notation, we call that energy  $Q_{dc}$  where the  $c$  stands for consumption. Then the total resources that return to the Supply reservoir are denoted by  $Q_{sc}$ .

## 5.2 Consumption

As we mentioned in our survey of previous work, thermoeconomics has tended to deal with production to an almost total exclusion of the importance of consumption[3]. This was mainly due to the fact that the supply determined prices by the reduction of entropy content they achieved. Therefore, consumption becomes an afterthought, an inevitable byproduct of more fundamental concepts.

However, there is no economic reason for that to be true. Both supply and demand interact to set prices [6]. A physical model that does not take into account that fundamental economic truth cannot be truly correlated to actual economic models. Indeed, a system with only a Carnot engine consuming energy will run out and collapse.

Furthermore, ignoring the use of the fruits of production is a mistake. The economy is fundamentally a cycle that feeds on itself in order to create and organize wealth in a society. And this cycle is supposed to be virtuous (if we take into account the side effects of economic activity like pollution and overwork and compensate adequately for the damage they do to the fundamentals of the economy). Part of the total sum of production is of course consumed, and the rest is invested. However, market economics assumes that all production finds a buyer thanks to equilibrium pricing[6].

### 5.2.1 Consumption in a Thermodynamic Perspective

We can see production as a thermal phenomenon. In our model, consumption is represented by the use of work to cool the demand reservoir. Therefore, work is 'making' workers pay for their consumption. In a deep sense, that is the purpose of production; goods and services compel workers and owners to give up their money and capital to purchase them.

To model such an economic process of consumption (which in our usage includes investment goods), we run a heat engine in reverse. This concept is reinforced by the thought of the economy as a cyclical machine. Only something running in the reverse direction as production can allow the long term sustainability of the economy.

Indeed, the addition of the refrigeration cycle in the heat engine solves the problem of wasted energy arriving into the cold reservoir. The rents and wages to compensate the inputs of production are removed from the demand reservoir by the goods and services. Of course, this is a cumulative process.

And in our model, as in real life, the cyclical nature of the economy is critical. However, if these two components of the complete cycle were the sum-total of the system, there would be stagnation. The system would be caught in an infinite loop with no prospect for improvement in the situation of the economy. No economic growth would be possible; a consequence, of course, of the Law of Conservation of Energy. To overcome such a stringent requirement, modifications have to occur in the model. There should be multiple ways to get around that requirement. We have chosen to use just one, the concept of the Value Multiplier.

## 5.3 The Application of the Heat Engines

Up to now, we have considered the heat engines as production mechanisms. We have talked about the general nature of a heat engine and its basic components. However, to make more detailed calculations, and eventually predictions, using the model we must delve deeper into the specific kind of heat engine we are considering.

A heat engine is defined by the thermodynamic cycle it obeys. The thermodynamic cycle, in physical terms, is the means through which a thermal system extracts work energy from the heat energy present in the particles in the system. It is generally expressed through graphs, such as a Pressure-Volume graph of a Temperature-Entropy diagram. To avoid the complications in defining in specific terms Pressure and Volume in our economic analogy, we will focus on the Temperature and Entropy concepts.

The cycle a heat engine obeys determines its efficiency: the percentage of work it can extract from a given amount of heat energy. In our case, we have assumed throughout that our model will have the maximum efficiency that is theoretically possible. The thermal cycle that fits that description is the Carnot cycle, with efficiency

$$\eta = 1 - \frac{T_d}{T_s}, \quad (5.1)$$

where  $T_d$  is the temperature of the colder reservoir and  $T_s$  the temperature of the hotter reservoir. As a bonus,  $\eta$  depends only on temperatures, avoiding the more problematic situation of defining pressure and temperature. Since a refrigerator is a heat engine in reverse, we can use the Carnot cycle in reverse to define it.

As an observation, we emphasize that reservoirs contain energy, only inasmuch as the particles within it have heat energy. The temperature of the reservoir is a macro measure of the aggregate micro heat in the ensemble of particles that comprise it. Indeed, temperature is an intensive measure, while energy is extensive<sup>1</sup>. If we extend our analogy of energy as a numerical value on the amount of resources,

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<sup>1</sup> Extensive quantities grow along with the size reservoir, while intensive quantities do not. If we double the size of a reservoir with a given temperature, the energy will double without altering the temperature. In that sense, it justifies our calling temperature an energy density in the same way that the density of a material does not change when you add more of it.

temperature becomes a density of resources, or a kind of average wealth. And as those particles do work in the heat engine, they lose heat energy (or resources) to the production mechanism.

We note, too, that these heat engines are used to justify the application of thermodynamics, and therefore statistical mechanics, to economic systems. If we can define a more macro approach to economics, it is easier to justify the statistical approach used in more small-scale descriptions.

### 5.3.1 The Short Term Model

The model as we have described it thus far works as follows:

1. Resources (Capital, labor, land, knowledge, etc.) leave a 'hot' reservoir.
2. By a process (such as a Factory) they become a product.
3. The exhausted factors of production go to a 'cold' reservoir.
4. Those factors (or particles in the physical sense ) are heated by the products.
5. That consumption reflects the value and then (with the value multiplier) can add some value to the total amount in a compound process.

We call this description the short term model because this cycle is true in the short term. Economic growth alters this, but only in a larger focus that we have not yet considered as part of the model.

## 5.4 Consumption and Increasing Production

However, let us define the value multiplier  $V$  (for reasons that will become apparent later) as a factor that changes the thermodynamic work done by the heat engine. The new effective work is the one that goes into the refrigeration cycle. Since the multiplier increases the total amount of work, which is energy without entropy, the net effect of its existence is to increase the total amount of energy (or wealth) in the system without a consequent increase in entropy. Over the long term, this would increase the system's total wealth.

However, since this is a Carnot cycle, a relatively simple calculation would lead us to find that after a engine-refrigeration, or production-consumption, iteration there would be a net energy flow from the demand reservoir and into the supply reservoir of  $(V - 1)Q_{dp} = (V - 1)(T_d/T_s)Q_{sp}$ . This is in addition to the new energy created by the value multiplier itself, which would be the difference between the total heat energy gain by the supply reservoir  $\Delta Q_s$  and the total heat energy loss by the demand reservoir,  $\Delta Q_d$ . In what follow we will calculate these quantities, but we can say (anticipating the results) that the increase in total energy  $\Delta Q_V$  will be

$$\Delta Q_V = \Delta Q_s - \Delta Q_d = (V - 1)\eta Q_{sp}, \quad (5.2)$$

where we use  $\eta$  as defined in equation 5.6. Now, for an example in how to calculate using our proposal, we will show the way the energy flows work.

One of the fundamental characteristics of the Carnot cycle is the ratio between heat flows entering and exiting and the temperatures of the reservoirs. If we label the hot reservoir  $H$  and the cold reservoir  $C$ , then

$$\frac{Q_H}{Q_C} = \frac{T_H}{T_C}. \quad (5.3)$$

In our case, we have two Carnot cycles. Because of conservation of energy, we can state that the work, and the heat flowing in and out of the reservoirs must add up in an accounting of energy so that

$$W + Q_C = Q_H. \quad (5.4)$$

The only difference between the engine and the refrigerator is that all signs of heat flow are reversed.<sup>2</sup> Now, if we use the two previous equations right, we can say

$$W = \eta Q_H = (1 - T_C/T_H)Q_H, \quad (5.5)$$

which will hold true for both the engine and the refrigerator, after we define

$$\eta = 1 - T_C/T_H. \quad (5.6)$$

Let us apply these concepts directly to the system we propose.

For the engine, production, part of the process, since the hot reservoir is the supply reservoir,

$$W = \eta Q_{sp}. \quad (5.7)$$

. However, the same idea should apply to the refrigerator, but with the change that instead of  $W$  on the left side, we have  $VW$  ( $\eta$  should stay the same, there being no reason for it to change since it merely depends on temperatures which we assume are unchanged) so that when we change  $Q_{sp}$  to its consumption equivalent  $Q_{sc}$ ,

$$VW = \eta Q_{sc}. \quad (5.8)$$

Solving for  $W$  and dividing the resulting equation by each equation 5.7, we find

$$Q_{sc} = VQ_{sp}. \quad (5.9)$$

Taking into account the different directions of these energy flows about the supply reservoir, the net change in heat energy after a production-consumption cycle will be

$$\Delta Q_s = Q_{sc} - Q_{sp} = VQ_{sp} - Q_{sp} = (V - 1)Q_{sp}. \quad (5.10)$$

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<sup>2</sup> As mentioned earlier, this is the reason why our proposal is not a perpetual motion machine. The reversibility of the Carnot cycles makes it so.

The same will apply to the energy flow to and fro the demand reservoir, but in the opposite direction. Using equation 5.3 we see

$$\Delta Q_d = Q_{dc} - Q_{dp} = (V - 1)Q_{dp} = (V - 1)\frac{T_d}{T_s}Q_{sp} \quad (5.11)$$

So, the Value Multiplier manages to both create wealth and to extract further resources from the particles. So, the thermodynamic efficiency is boosted as well. One way this process could manifest itself in economic terms is the better return on a worker's time. This would manifest itself in higher wages for workers, if the market assigns wages in accordance to value of the work done. Therefore, the Value Multiplier becomes one way the model represents increasing productivity of labor.

In a way, this increasing productivity is logical. Workers tend to produce more when they are healthy, as Nazi Germany found [12] in its use of slave labor. So, in a sense, economic progress as represented by the Value Multiplier and its compound accumulation both boosts the wealth of the economy, and allows us to harness it more effectively<sup>3</sup>. This can be seen in the formula for effective work,  $V\eta Q_{sp}$ . The first term is the Value Multiplier itself, the last is resource usage in the aggregate. The second term is, of course, the one that will be affected by the shifting of energy from the demand reservoir to the supply reservoir.

### 5.5 The Value Multiplier

We have mentioned that conservation of energy prevents wealth creation in our analogy. And if the Value Multiplier is equal to 1, then the mechanism described in the previous section doesn't work either; production would stagnate. So we need a Value Multiplier that creates wealth, larger than 1. Indeed, we find that wealth has

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<sup>3</sup> The same caveats due to externalities that are not priced in, such as the environmental damages we do, apply

been created throughout human history. So there must be some economic growth and the model should reflect that.

We must define the effect of the Value Multiplier. The compound effect of cumulative value multipliers creates something that we believe can be related to the economic concept of the Value Multiplier; a reflection of the constant addition of value in human activity. Basically, the Value Multiplier is the ratio between the price of something, and its cost. As we know, prices are set in the marketplace by the interaction of the supply and the demand. In general, the higher the utility (or usefulness) of a good, the higher the price it will command in the marketplace. But, in order to create a greater Value Multiplier, not just a greater price, the utility of the good produced must be greater, not just than the competition's, but greater than the utility of the resources that went into the production process. The utility of the labor done, of the raw materials used, or the energy consumed must be smaller than the product's.

In a way, the arguments in [3] about low entropy being the determiner of value are reversed. The higher the Value Multiplier, the more additional energy is introduced into the system without an accompanying creation of entropy. This means that high  $V_i$  can reduce the entropy of the system. With lower entropy, more work can eventually be done by the system; more production can take place. In a sense, low entropy is not the mark of value, it is what high value achieves. The better the products are, the more we are willing to pay over the cost of production. And better products tend to have a higher demand, which means they are more desirable. In theory, more desirable goods should make us happier [6].

Indeed, should the Value multiplier be smaller than 1, then there would be net flow from the supply and into the demand. Without a temperature difference, the heat engine shuts down. Translated into economic terms, the economy would shut



down, as it must do if its economic activity made products less valuable than the resources needed to make them.

The aggregation of so many Value Multipliers along the whole history of the system is what we call economic growth. Indeed, the economic growth we find is by definition built on the success and achievements of previous periods in history. And this increase in aggregate utility is reflected in the fact that the Supply reservoir receives more energy (or wealth) that it put into the process. Essentially, as the products become resources (because consumption restores and boosts labor productivity after it does its part in the production process) they increase the total amount of resources available for production. Therefore, the temperature of the hot reservoir increases, and the efficiency of the total system goes up.

If we were to calculate such an increase in efficiency due to the system, we would find that the change in efficiency can be written as

$$d\eta = \frac{T_d}{T_s^2}dT_s - \frac{dT_d}{T_s}. \quad (5.12)$$

We can show this, if we take the derivative of equation 5.1 with respect to  $T_s$ , which is

$$\frac{d\eta}{dT_s} = \frac{T_d}{T_s^2} - \frac{1}{T_s} \frac{dT_d}{dT_s} \quad (5.13)$$

and multiply by  $dT_d$  both sides. Since energy is flowing into the supply reservoir and out of the demand reservoir,  $dT_s$  must be positive and  $dT_d$  must be negative. Therefore, the change in efficiency must be positive. This effect, caused by the Value Multiplier, is the easiest to understand due to its thermodynamic nature.

**The Value Multiplier is a function of the perceived value. It is set by (we assume) perfect markets, and is regulated by supply and demand.** We assume that these markets are self correcting, meaning that they react quickly to changes in conditions and restrictions. Hence, prices levels adjust quickly and are not 'sticky'. Of course this is true in theory and not necessarily in reality. If we were to be strict,

this is also true of thermodynamics, where the assumptions made are not necessarily strictly applicable in real situations. Indeed, market failure can affect deeply the functioning of the ideal model. One such example is health care spending, a critically important part of the economy. [13]

## 5.6 Short and Long Term Perspectives

Upon close observation, the thermodynamic aspects of the model stand out as different from the Value Multipliers. Since the Value Multiplier is the source of long term economic growth, in some ways it must be more stable. An example is Puerto Rico's economy, where trend growth, around which economic conditions fluctuate, has been quite stable for periods of decades, with one significant inflection point in 1974 [11]. Indeed, if the Value Multiplier is representative of our capacity to innovate, one would assume that it is quite stable, because the basic abilities of the human being should not change along with the business cycle. And if we take away the effect of the Value Multiplier, we end up with mostly thermodynamic fluctuations, which we would then have to postulate represent changes in the business cycle part of the explanation for GDP.

One place where our model could be examined is the role of inflation. We must assume that energy represents value in real, not nominal, units of money, so that the model can take energy to be wealth. Hence, inflation and deflation cannot be considered part of the model in that sense, since we eliminate from consideration that part of the economy from the energy sector. However, we know that those concepts can deeply affect economic growth. The key here, of course, is to recognize that inflation, deflation and monetary policy in general affects financial markets directly, and only indirectly the 'real' economy. This is not to minimize its effect, since finance can cripple the economy as we have seen in the past few months, but only to emphasize that the financial system is separate and will not be considered a

normal industry. Indeed, we should expect financial systems to oscillate more wildly in terms of economic behavior than GDP (the 'real' economy).

## CHAPTER 6

# THE VALUE MULTIPLIER IN DETAIL

The distinction between long and short term behavior has been discussed before. However, to make sense of the long term behavior, we must find an explanation for the trend. Economics has recently focused on Endogenous Growth Theory as a possible way of identifying causes for long-term growth. We will later find that a case can be made for putting the trend behavior within the Value Multiplier's purview.

### 6.1 Endogenous Growth Theory

EGT has been spearheaded by Paul Romer, an economist who believes in changing the classical catalogue of economic resources. Historically, resources have been placed inside three broad categories: land, capital, and labor. However, in his view it is necessary to explain the divergence in economic growth in the midst of the industrial revolution. The explanation cannot be found in the increase in resources, since they were broadly similar to those available beforehand. What Romer says made a difference is the way those resources were used. In a review article, he makes an analogy to a kitchen, where as the cook learns more recipes, the value of the things he can make increases even though the ingredients did not change much.

This analogy illustrates the rise of knowledge. Indeed, the existence of the Industrial Revolution in the 19th century was due to increased efficiency in machinery, due in part to the existence of the steam engine and of coal as a source of energy. Past peoples who did not know of those possibilities could not make use of the coal and metals in their lands. Knowledge increased wealth.

In a similar way, lands where oil could not be extracted in previous eras saw their value increase. Knowledge, in this case geological, makes possible increases in the total wealth of the society. In the case of the steam engine, it was physical knowledge; in the case of medicine's increased effectiveness, it was biological information. This knowledge is a source of wealth, yet it is intangible. But its effect is evident in our modern world.

Indeed, in the conception of Romer, knowledge is especially valuable because it is multiplicative. Resources are used for one or another purpose, and that use precludes its further use. However, knowledge spreads in multiplicative fashion, much like gossip (which is a form of knowledge, however inaccurate it may be). This increase is also cumulative, for previous advances in the scope of the known increase the scope of the knowable. Indeed, an analogy can be made to compound interest. In fact, the Value Multiplier acts in a similar manner. When the Value Multiplier appears, it increases the amount of energy available for the next round.

We must contrast this with the traditional econophysics view described by Georgescu [3]. In that older model, energy and resources was seen as a strong contribution to economic growth. Value was simply the incorporation of those resources (inputs) into goods (outputs). Value was conserved, and simply reshaped. One example of such thought was produced by the hunt for resources that Europe engaged in during the colonial period, and beyond. Nationalists through those countries postulated a theory that if only the Europeans would leave, the resources being stripped from those countries would benefit their inhabitants. Independence was sold as a way to take the value of those resources for themselves, because they postulated that Europe's wealth came from exploiting them. However undeniable the colonial exploitation, the truth is that Europe has continued to boom relative to those countries, mostly because of the Value Added by their production to the raw materials they import. Europe added value to resources; many former colonies

have become rich by selling their resources like oil, but as soon as its price drops or supplies run out, their economies collapse. This begs the question: what is the Value Multiplier? Indeed, the Value Multiplier is a set of intangibles that contribute to the value of an object. It does not have to be scientific knowledge. It can be simply knowing the market, a bit like Apple. It is difficult to see how the iPod incorporates any new kind of scientific knowledge. Beyond the packaging and programming, the iPod is no great leap in technology, and it has not been marketed as such. However, it commands high prices, and as any trip through electronics store can attest, higher prices than devices with comparable technical profiles. The Value Multiplier reflects that, and goes beyond simple physical-material conception of economics, and to a more modern conception of knowledge and intangibles as a source of growth. In fact, the cumulative nature of knowledge is critical to economic growth, because knowledge can be shared. Indeed, patents exist as a way to share knowledge, since they are public, without destroying the value that comes from having worked to discover it.

### **6.1.1 Where does that knowledge come from?**

Fundamentally, knowledge is a human creation. Our existence, which in itself is a reduction of entropy, is a consequence of the Sun's light. Without the sun, life would never have appeared. As a source of energy, it had the result of decreasing our own entropy, here on Earth, and promises to be the source of further reduction in the foreseeable future and beyond. Even our most basic nutritional needs are fulfilled only because of the Sun.

All the potential that humans may have for ordering our planet in different ways is subject to our own interpretation. Practical considerations preclude us from measuring the entropy of every living being here, and of our products. The theories we mentioned in our chapter on Previous Work are all dependent on entropy reduction as the source of value. However, it is impractical to do so on a large

scale. Even if we were to grant the truth of those theories, we could not truly talk about entropy-based values, because we could not calculate it. Hence, even in that eventuality would force us to depend on the market for estimations of value.

If we go to a temperature-entropy diagram of the Carnot Engine, we see that the lower entropy eventually goes in the engine, the more efficient the engine will be. The use of resources in production is more effective when entropy goes down. So both entropy minimization and work maximization are identical in effect. Since value in economic terms increases monotonically with utility, entropy minimization is creating value. However, since the market is the way we measure value, it is more proper to say that perceived entropy reduction is the source of value.

As a final note on this topic, we must emphasize the context of our proposal in the micro/macro dichotomy. Even though our model may be more suited to microeconomic situations than a total macroeconomic picture, we use EGT as a way to illustrate the relationship between them. The relationship between microeconomics and macroeconomics is tenuous and poorly understood. This section seeks to find a place where our model can bridge the gap between concepts in both scales of economic analysis. We do not pretend to solve this problem, only to connect in a manner analogous to what has been done in Physics.

## **6.2 Profit**

Profit is the difference between cost of production and price. Prices are set in markets, and therefore profits are in part determined by them. The stakeholders of production receive the benefit of those profits. Of course, we assume throughout perfect markets.

Finance is concerned with redirecting accumulated money into enterprises that may be profitable. Different instruments, such as stocks and loans and bonds exist for that purpose. The rent for the use of that capital is taken from the profits of the production mechanism. Stocks, being stakes in the ownership of the enterprise,

are priced not only relative to present profits, but should take into account future profits. Those future profits are unknown, so they must be projected by the investor.

The higher both the present and expected profits are, the more valuable a stake in the company can be and the higher stock prices go. The price of a bond, too, reflects both present and future circumstances and their effect on the company's ability to repay their obligations. And the people who put up capital can expect dividends and interest payments as well. Those are determined by the market rate of return (what a risk free investment would receive) and the expected risk. The operative word throughout is expected. In a fundamental way, it is all a bet on the future performance of the investments made. The more accurate the perception of the investor, the more he should make. <sup>1</sup>

What we can take away from this whole discussion is the fact that people expect to gain money from their investments, to make a profit. This profit is of a different nature from the profit made from production, but it is still increase in wealth. We could then define a Value Multiplier, since investment 'creates' wealth. This is a relatively undeveloped idea in the model, yet it could increase our understanding of financial systems within, and its effect on the economy. As we have seen recently, finance can be both the cause and the slayer of economic growth.

### **6.3 Derivatives and the Current Crisis**

As Keynes said, projections of projections of human behavior are risky. And gaming the stock market depends on the projection of projections of profits that depend on the value people put on the goods produced. So, in a sense the investment Value Multipliers, should it exist, would depend on other Value Multipliers. This

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<sup>1</sup> This of course, ignores short-term fluctuations that can change completely the equation. The best cards player, with the odds in his favor, can be driven by bad luck to bankruptcy regardless of his ability.



nesting is of course unstable. Should the foundations be disturbed, the whole edifice built on top could collapse, reducing the wealth in the economy in uncertain ways. In addition, a collapse in the credit markets would be terrible. Derivatives, whose value is based on the value of their underlying securities, which are valued according to the profits expected of actual enterprises, are even more unstable. That instability could create a perennial boom and bust economy in our lifetime.[14] That financial collapse would cripple the financial mechanism that boosts  $T_s$  by reducing  $T_d$  by means of investing savings. That would reduce the thermal efficiency of the main production mechanism and the total production as well.

In the Next chapter we shall establish a way of calculating work done (GDP in economic terms) and find that the Keynesian Multiplier can be found here, along with the way financial systems increase production.

## CHAPTER 7

# DETAILED CALCULATIONS AND THE KEYNESIAN MULTIPLIER

To begin our application of the model to economic systems, we must describe in greater detail the way our model works in the production process. Once we describe the heat engine and its cycle, we can then establish the way we can establish analogies to a key concept of economics, the Keynesian Multiplier. Before we enter further into the calculations, we must make clear that, although we have outlined a relationship between microeconomics and macroeconomics in the framework of this model, we tend to believe that this model as it stands is easier to relate to microeconomic systems. However, the use of macroeconomic concepts such as fiscal stimulus is merely directed at hinting at a relationship between the two, and at the effects of that stimulus on a microeconomic system.

Basically, the Keynesian Multiplier is a way of describing how fiscal policy, namely government deficits, can alleviate economic distress by making up the gap between actual production and the full potential of the economy to produce. The gap between the two is a consequence of slowdowns, and results in increased unemployment. [15] [16] Since economic failure can be self-perpetuating, it is occasionally convenient to incur in deficits to stop a cycle. Such spending helped stop the Great Depression, and when it was reduced in the midst of an incipient recovery economic growth was retarded severely [17]. We should be able to see within the system the mechanism through which deficit spending could influence short-run economic growth.

## 7.1 The Thermal-Economic Cycle

The behavior of the production-consumption can be divided into a series of steps.

1. Resources (Capital, labor, land, knowledge, etc.) leave a 'hot' reservoir,  $Q_{spi}$ .
2. By a process (such as a Factory) they become a product  $W_i = \eta_i Q_{spi}$ .
3. That product has a different value than the value of the resources,  $W_i \rightarrow V_i W_i$ .
4. The exhausted factors of production go to a 'cold' reservoir  $Q_{dpi} = \frac{T_d}{T_s} Q_{spi}$ .

We have mentioned before that GDP should be broadly equivalent to economic output, and that economic output in the model should increase when  $\sum_i W_i$  increases. Since  $W_i \propto Q_{spi}$ , the more resources are consumed, the more output there is as a rule of thumb. However, efficiency is important as well since inefficient production is harder to sustain and the wasting of resources is a drag on the long-term health of the economy (by reducing the amount of resources available for the future) and on the health of the planet (through pollution).

After the production process occurs in the heat engine cycle, there is a series of steps designed to represent the consumption process, which we have modeled as a refrigeration cycle.

5. Those particles are heated by the products  $Q_{dc} = V Q_{dp}$ .
6. That consumption restores their value and adds some, which is deposited into the hot reservoir  $Q_{sc} = V Q_{sp}$ .
7. The energy contained in the hot reservoir changes by  $\Delta Q_s = (V - 1) Q_{sp}$ .

A key takeaway here is the importance of the Value Multiplier. Since it modifies the output of the economy without involving directly either resource usage or thermal efficiency (although we cannot say that  $V_i$  does not depend on the mix of resources used and may be subject to diminishing returns), a sustainable path to increased economic growth is focusing on the concept of adding value to resources. This adding of value must have, then, a reason to make it happen. Using our model

to explain a particular economic phenomenon should illuminate the basis for evolving Value Multipliers. And to calculate and use the model in the context we want, we must be able to make predictions.

It is possible to show that in the absence of multiple supply reservoirs (and their competition) we cannot make predictions. The Value Multiplier must be an extraneous parameter. The reason this occurs is that a single reservoir cannot achieve equilibrium since it does not compete with anything. Therefore, every attempt to equal the derivative of economic output to zero and optimize it results in the second derivative being zero as well. The addition of more reservoirs allows equilibrium to occur and delineates the requirements for it to be either stable or unstable.

The fact that competition is essential for us to be able to calculate in the model sheds light on the system. Macroeconomics is aggregate behavior, the result of the addition and interaction of many, many markets. However, the individual markets must be ultimately traceable to the fundamentals. Since we need competition to propose a model with non-extraneous Value Multiplier, the interaction between the various individual markets must be the source of  $V_i$ .<sup>1</sup> We must find a way to model competition.

## 7.2 GDP and Unemployment in our model

GDP is the sum of all the production in the economy, where production is measured through the multiplication of price and quantity of every good and service. In our model, that would be  $\sum_i V_i W_i$ . In economic lingo, it is the aggregation of every production mechanism in the economy. Similarly, we could define Unemployment as the underuse of a particular resource, namely labor. We assume that an increase in unemployment is correlated to a reduction in output. In that case the thermal efficiency goes down, taking with it the output. In other words, as  $\eta = 1 - T_d/T_s$

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<sup>1</sup> We use the index  $I$  to indentify the reservoir we speak of.

approaches one, more of the resources being used to produce are effectively turned into work, and less are left unused (or wasted).

It is important to note that the Value Multiplier is unrelated to the thermal efficiency. That efficiency merely measures the proportion of the resources available that can be turned into the desired product. This should not, in theory, affect the Value Multiplier because it is a characteristic of the product, not the amount sold. Of course, in this discussion, higher-order considerations (such as a potential bandwagon effect when people imitate other's consumption patterns) will not be taken into account. The principle we want to emphasize here is that unemployment (a key economic indicator) is to be related to lower thermal efficiencies.

A key point on the Value Multiplier bears repeating: without it, no true economic growth (or more specifically, productivity growth) would be possible. If the Value Multipliers equaled one, aggregate global wealth could not increase. Maximum usage of resources would be our economic growth constraint. No increased value would be possible without increasing resource usage. Needless to say, we do not observe such behavior. One example could be increased nutritional knowledge, which increases wellbeing without necessarily increasing consumption.

### 7.2.1 Optimization and Constraints

When we disregard the effect of increased resource consumption in the model, we can postulate a series of constraints, which are needed to calculate. We assume finite consumption of resources. Therefore, we fix  $\sum_i Q_{spi} = Q_{sp}$ . There will be competition for the use of those resources. The relative distribution of those resources must be a consequence of the model, since we have always held optimization of aggregate utility to be the deciding factor in consumption. But to find those relative distributions, we need to determine the controlling factors.

We have used endogenous growth theory before to justify some of our concepts. The theory assumes maximization of profits (for firms) and utility (for individuals).

Profit is the difference between the price and the cost of production. It is simply  $Q_{sci} - Q_{spi} = (V_i - 1)Q_{spi}$ . Utility determines the value of a product. Since relative price is determined by relative utility, we write utility  $\propto \sum_i V_i W_i$ . The maximum expenditure (of resources) is our constraint. To that end, we seek to maximize utility within that constraint. Under these assumptions, we can do some preliminary calculations.

We note, however, that the temperature of the reservoirs will change due to energy flows. The ratio between energy flow and temperature change is called the heat capacity. We have to estimate the heat capacity. We assume our system to be reversible, and reversible processes can be inverted to return to the original state. Among the properties of such systems,  $\sum_i \frac{\Delta Q_i}{T_i} = 0$  for all the reservoirs. However,  $\Delta Q_i = C_i \Delta T_i$  by definition. Therefore  $\sum_i \frac{C_i \Delta T_i}{T_i} = 0$ . Solving for the heat capacity of one reservoir,  $C_j = -T_j \sum_{i \neq j} \frac{C_i \Delta Q_i}{T_i \Delta Q_j}$ .

Heat flows from one body to the other. The ratios of heat flow will then be negative. This means that the sum has a negative sign. Customarily,  $C_j$  is a characteristic of the body  $j$ . We assume that  $C_j$  does not depend on the characteristics of other bodies. Therefore, we can ignore the properties of the other bodies and approximate  $C_j = C_j(T_j) \approx A_j T_j$ .

### 7.3 The Keynesian Multiplier

The Spending Multiplier is a consequence of Keynesian economic thought. Basically, it represents a larger increase in total consumption from a change in autonomous spending. The whole point is that government expenditure of 100\$ can increase total spending by more than that. Our model predicts that consumption be proportional to  $V \frac{T_d}{T_s} Q_{sp} = V(1 - \eta) Q_{sp}$ . Increases in spending, proportional to  $Q_{sp}$ , are multiplied by  $V(1 - \eta)$ .

We can see from the previous paragraph that higher thermodynamic efficiency results in the lowering of the multiplier. We have seen that higher  $\eta$  represent

increased tapping of capacity. The spending multiplier in our model is higher when capacity usage is lower. This is similar to what macroeconomics has to say about autonomous spending increases. One such increase is a fiscal stimulus package, which can boost the economy when low usage of capacity occurs. <sup>2</sup>

### 7.3.1 The Financial System

There may be a relationship between the unexplored financial reservoir and the system that makes the efficiency go down. Deferral of expenditure by the demand results in capital accumulation and investment in supply production. In such a model, the average rate of return could be a financial Value Multiplier,  $V_f$ . That  $V_f$  depends on the expected profits of the reservoirs. Those profits are determined by  $V_i$ . Such investment increases  $T_s$ , and decreases  $T_d$ , and therefore increasing  $\eta$ . In perfect circumstances, a financial system would increase the maximum efficiency, optimizing resource usage by harnessing unused resources as investment.

By increasing the efficiency,  $V_i\eta_iQ_{spi}$  or GDP is boosted. However, when  $V_f$  fluctuates (as it is bound to do), then things could change. When Value Multipliers decrease, the net flow into the hot reservoir decreases as well, because net flow equals  $(V-1)Q_p$ . This Financial Value Multiplier we are proposing is based on perceptions, as is every other Value Multiplier. Increased risk makes investors demand higher rates of return. If effective interest rates were increased, the demand side would have decreases its energy outflow, boosting  $T_d$ . Therefore, in such circumstances,  $\eta_i = 1 - T_d/T_s$  would decrease.

This would decrease the percentage of usage. Those interest rates increases could be ordered by central bank or an effect of bubbles bursting. For example,

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<sup>2</sup> Of course, this is different from spending on things such as education that are more properly called investment. Such spending increases the general store of knowledge and should affect the Value Multiplier as well.

if effective interest rates were increased, the demand side would have more money flowing to it and would increase  $T_d$ . Therefore,  $\eta_i = 1 - T_d/T_s$  would decrease. A lot of business spending is done via loan, and presumably most of it is for maintaining inventory. If maintaining inventory gets harder, less of the economy's capacity would be tapped. Therefore, fiscal stimulus could be needed, to increase  $\eta$ .

Such a decrease in  $\eta_i$  would decrease GDP. A large amount of business spending is done via loan, and presumably most of it is for maintaining inventory. If maintaining inventory gets harder, less of the economy's capacity would be tapped. Therefore, fiscal stimulus could be needed to pick up the slack. If resources are injected to the supply reservoir,  $T_s$  increases. Government intervention could easily increase  $\eta$ . The multiplier of such intervention would be higher. However if the problem is not enough wealth creation, then  $V_i$  must be addressed. Of course, increasing  $V_i$  is a different problem than fiscal stimulus.

#### 7.4 Calculation and the Heat Capacity of the Reservoirs

There is a further question for the model, which has to do with heat capacity. Heat Capacity is defined as the heat needed to change by one degree the temperature of a body. The heat capacity of the reservoirs is therefore important to make any kind of prediction with the model. To do anything of note with the system, to calculate in any way, we need the heat capacity. Once we have it, we can take the net heat flow in every cycle of the system and divide it by the heat capacity. We sketch here an attempt to grasp at its true form.

If we assume that the total change in entropy for the system is equal to zero, which is basically the maximum efficiency condition, we find  $dS_{s1} + dS_{s2} + dS_d = 0$ . This is an approximation on our part to try and find a heat capacity relationship, as it is not necessarily true that entropy change is zero, as it could be more or less than zero depending on the size of the Value Multiplier. This entropy condition is a good approximation for the short term. Since we know that  $dS = dQ/T$  we can



write

$$dS_d + dS_{s1} + dS_{s2} = \frac{dQ_d}{T_d} + \frac{dQ_{s1}}{T_{s1}} + \frac{dQ_{s2}}{T_{s2}} = 0. \quad (7.1)$$

We remember that  $dQ = CdT$ , where  $C$  is the heat capacity, so the condition for constant entropy can give us an expression for the heat capacity of the Demand reservoir, which becomes equation 7.2.

$$C_d = -T_d \left( \frac{C_s}{T_{s1}} \frac{dT_d}{dT_{s1}} + \frac{C_s}{T_{s2}} \frac{dT_d}{dT_{s2}} \right) \quad (7.2)$$

We know that the heat capacity must be positive by definition. The negative sign in equation 7.2 cancels with the sign of the derivatives, because in general, as the supply temperature moves in one direction, energy will be transferred from one reservoir to the other, so the demand temperature will move in the opposite direction. The justification for this can be found in the Carnot cycle, where our efficiency is determined by equation ???. We can write the temperature  $T_{si}$  in terms of efficiency as

$$T_{si} = \frac{T_d}{1 - \eta_i}. \quad (7.3)$$

From here we can calculate the derivative of the supply temperature,

$$\frac{T_{si}}{T_d} = \frac{1}{1 - \eta} \quad (7.4)$$

If we assume that efficiency is constant in the short run, we can say that the derivatives are constant. And if we make the further assumption that heat capacity depends only on the temperature of the reservoir itself, and not on extraneous factors such as the temperature of the other reservoirs, we can write the heat capacity as in equation 7.5.

$$C_d = \alpha T_d \quad (7.5)$$

The same will be true for the supply reservoirs: the heat capacity would be proportional to the temperature. We shall deal with this preliminary calculation in the final chapter.

## CHAPTER 8

# CONCLUSIONS AND RECOMMENDATIONS

As this work draws to a close, we want to highlight both what we believe are the main areas covered and the questions they raise. There are three such points of concern. First of all, the relationship between microeconomics and macroeconomics. Second, the impact of fiscal stimulus on microeconomic systems. And third, the business of extracting numerical results from the model.

### 8.1 Microeconomics and Macroeconomics

The two main branches of economics differ as to the scale of the phenomena that they model. We are conscious that there is no consistently effective way to establish the correspondence between them. However, physics is not lacking in examples of theories that describe the same fundamental behavior, but on different scales; scales so different that the mathematical and axiomatic framework that undergirds them would seem to be almost incompatible. Even more important, before the pattern of correspondence between the systems was discovered, they seemed to be incompatible. With an eye to this parallel, we examine some correspondences between micro and macro theories in physics.

Newtonian mechanics was, for centuries, the premier physical theory. However, the discoveries of many unexplained phenomena in such apparently unrelated areas as the light spectra of the sun, the color of objects as they become hotter, and the creation of voltage as metals are illuminated with light, led to theoretical research directed at explaining them. Curiously, from our perspective, although phenomenological explanations were quite useful (Bohr's atom, Planck's Distribution, Einstein's

Photoelectric Formula, etc.) these efforts had limitations. They could help explain and understand what was known, but there was an ad hoc quality about them. The insight to find a correspondence between the microscale phenomenological explanations and the macroscale Newtonian theory allowed them to fully understand the microscale and establish a solid footing for that theory. Indeed, the correspondence principle allowed them to test their assumptions and the logical consistency of their approach. And even more interesting, from our standpoint, is the effect the discovery of the microscale theory of mechanics, or quantum mechanics, had on thermal systems.

Thermodynamics, as it was conceived by the people who created it in the 19th century, was a science describing thermal events: the behavior of heat in physical bodies. The principles involved in it were also somewhat ad hoc, derived from observation and basic concepts such as conservation of energy. Indeed, thermodynamics made much progress by applying Newtonian mechanics to the microscale, such as the Maxwell distribution. Even so, it was the discovery of quantum mechanics that allowed physics to truly understand how particles interacted in thermal systems, and create statistical mechanics. Once that was done, it was relatively simple to find a correspondence between the new statistical mechanics and the thermodynamics that was already well known. That process of connecting was hurried by the fact that statistical mechanics tends to assume that particles do not interact, and that the predictions made using that assumption seem to be accurate. However, non-interaction can only take us so far.

A fundamental difference between these physical examples and the scale problem in economics is the huge complexity of human systems and social sciences. Assuming non-interaction is possible in particles, where in general the space between them is orders of magnitude larger than their own size. We can hardly say human behavior, and therefore economic behavior, can be perfectly predicted by a theory

that champions non-interaction. However, we can say that, if we can isolate the parts of the process due to interaction, and those that are not. In fact, we believe that macroeconomics should be much more about interaction, due to its sheer size, complexity, and non-linearity, than microeconomics.

Since we are saying, then, that microscale phenomena should show less interaction dependency than macroscale (because of a lesser degree of accumulation) then non-interacting theories should better predict microscale behavior. Indeed, we acknowledge the fact that it is relatively easier to model companies or businesses with our model than whole economies. However, one of the long term goals of this work is to find ways to link the two scales, to find a Correspondence Principle for Economics. We can note that our model, on the main, depended on the non-interacting theory of thermodynamics to describe the heat engines and the work they did, and the analogy to production they made. The one place where the laws of those physical, conventional non-interaction theories were suspended was the point where the Value Multiplier came into action. If we go back to the chapter that discussed the Value Multiplier, much emphasis was placed on the Endogenous Growth theory that describes knowledge as a foundation of the value of goods; that theory aims to describe macroeconomics. Indeed, Endogenous Growth theory helps put on a conceptual basis what is well known about humanity: that technology is the driving force in the betterment of our society and our understanding of nature, both our own and that of our surroundings[18]. The Value Multiplier is the place where, in a principally microeconomic description as this, non-interaction is gone, the human element of giving value to something is present; but it is also where macroeconomics has helped so much in describing a concept that is alien to a physical theory like thermodynamics, based so strongly on the conservation of energy that the Value

Multiplier ignores. We strongly believe that any link between macroscale and microscale economics that this model can help establish will be found in the Value Multiplier and its further conceptualization.

## 8.2 Demand-Side Stimulus

Another place where our model can illustrate things that are controversial in economics is demand-side government stimulus that stands in contrast to supply-side stimulus. In our model, although we have not gone deep into a comparison between the two, we can use some of our results to make an educated guess as to which side to stimulate. Using equation 5.12, which describes the differential of thermal efficiency when we change the temperatures. If we hold efficiency constant, and remember that  $\eta = 1 - T_d/T_s$ , then

$$d\eta = 0 = \frac{T_d}{T_s^2}dT_s - \frac{dT_d}{T_s} = \frac{1}{T_s} \left( \frac{T_d}{T_s}dT_s - dT_d \right). \quad (8.1)$$

Simplifying and substituting  $T_d/T_s = 1 - \eta$ ,

$$(1 - \eta)dT_s = dT_d. \quad (8.2)$$

Since we know that the efficiency cannot be larger than one, this equation tells us that a degree of temperature change in the demand is equivalent to a larger amount of temperature change in the supply. In other words, it is more efficient to stimulate demand, not supply. Note that we ignore in this formulation the heat capacity, which as we have seen and will see, has an effect on our model which we have not been able to fully take into account. In fact, the heat capacity could be a place where diminishing returns could have an effect. Let us illustrate a way that is possible. Heat change would be roughly equivalent to stimulus spending. If we assume that our calculation on heat capacity being proportional to the temperature of the reservoir are correct, we can substitute the definition of heat capacity,  $dQ =$

$CdT$ , into the previous equation, so that

$$(1 - \eta) \frac{dQ_s}{C_s} = \frac{dQ_d}{C_d}. \quad (8.3)$$

Substituting our formula for heat capacity,  $C = \alpha T$  where  $\alpha$  is a scale constant, we have

$$(1 - \eta) \frac{dQ_s}{\alpha_s T_s} = \frac{dQ_d}{\alpha_d T_d}. \quad (8.4)$$

Rearranging terms,

$$(1 - \eta) \frac{\alpha_d T_d}{\alpha_s T_s} dQ_s = dQ_d. \quad (8.5)$$

Again substituting the formula for efficiency into the temperature ratio,

$$(1 - \eta)^2 \frac{\alpha_d}{\alpha_s} dQ_s = dQ_d. \quad (8.6)$$

Therefore, even though our current estimate for heat capacity makes our case for demand-side stimulus stronger (since squaring a number smaller than one makes it even smaller, and  $1 - \eta < 0$ ) the scale ratio  $\alpha_d/\alpha_s$  complicates things. It could introduce a possibility of diminishing returns if the scale factor stopped being strictly a scale factor, and became a function of other variables. That could introduce some change into our system.

We must make a further qualification. As we mentioned earlier, the thermal part of the model (which is, in the language of the previous section, non-interacting) works in the nuts and bolts of the economy. Innovation should make itself felt in the Value Multiplier, and many who argue for supply-side economics believe that it is in innovation that tax cuts and related business oriented policies have their greatest effect. Nevertheless, this could be an area of fruitful study.

### 8.3 Computer Simulations of the Model and its Behavior

As we approach the end of this work, we seek to lay out some computational, numerical results. Before we do so, however, certain points must be clarified. Firstly,

we must acknowledge that our emphasis has been in providing the theoretical underpinnings for a more comprehensive approach. The practical application of our efforts must wait for further research, such as a comparison between an empirical area of the economy and a prediction from our system. Second, as has been mentioned before, the method of calculating the heat capacity is not obvious at present. Without it, any numerical results will be imperfect. Third, the Value Multiplier, by necessity, will have to be approximated since its exact form is another of the pending questions to be answered. All that said, we can present the result of some simulations in the hope of stimulating further research in this field.

To breathe some life into our system, it would not do to postulate a single supply and a single demand reservoir. It would merely result in a constant growth wholly dependent on the exact initial conditions, in particular the Value Multiplier. A way to change this would be to add another reservoir that could compete. Either a supply or a demand reservoir could be added. We chose to add a supply reservoir, mainly because we better understand its meaning and because we believe demand picks and chooses between the goods it seeks. In a way, we follow the reasoning behind the indifference curve: two goods compete, and consumers choose the basket of goods as their prices change, subject to a budget constraint. The indifference curve thus demonstrates the different combinations of the two goods that provide the same utility so that consumers are indifferent as to the choice. Here, the two goods are each related to a supply reservoir, so logically the next step is finding an analogy to the budget constraint.

The amount of goods produced, once acted upon by the Value Multiplier, is dependent on the thermal efficiency and  $Q_{sp}$ . A limit on the total amount of heat that can come from both reservoirs could be the equivalent to the budget constraint, and so we establish it. For comparison's sake, we used MATLAB to run a toy model of this concept, while postulating a constant Value Multiplier (in a sense ignoring



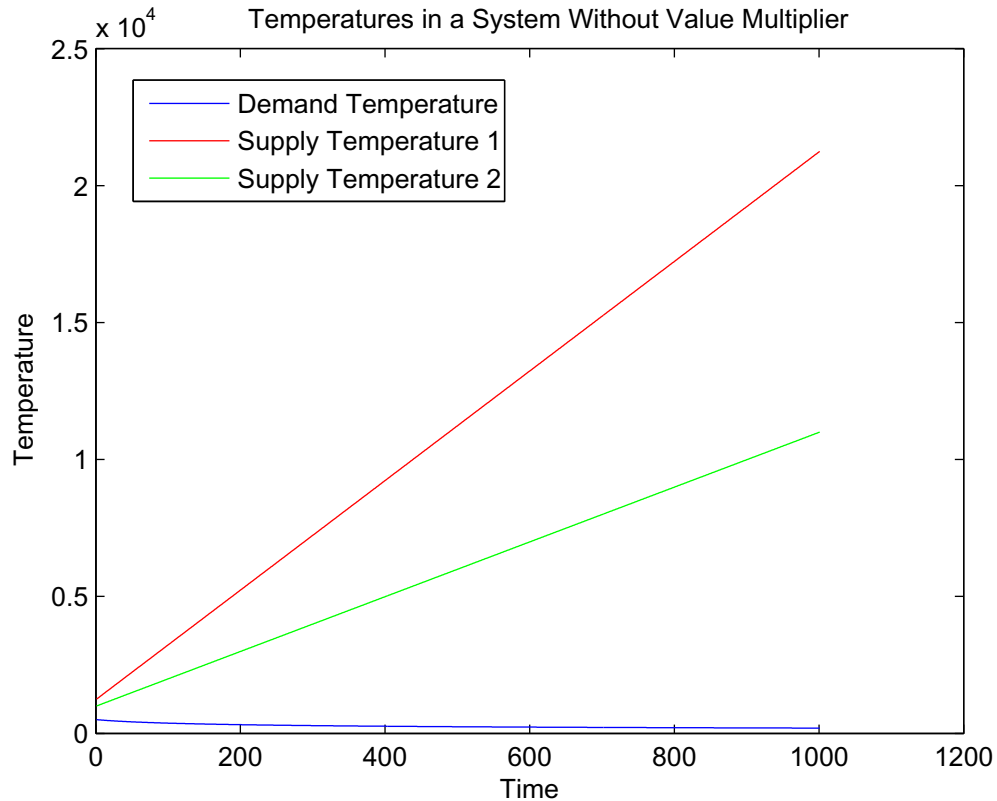


Figure 8–1: This is the Behavior of the Reservoirs when the Value Multiplier is ignored.

the effects of true competition). We assigned initial temperatures for the reservoirs, and used our result for the heat capacity (proportional to the temperature of the reservoir, and the scale factor is unimportant in the greater scheme of things)<sup>1</sup>. After each iteration of production and consumption for the reservoirs, we recalculate the temperatures using the heat flows and the heat capacities. The result is shown in Figure 8–1; the temperatures of the supply reservoirs increase over time, and the demand reservoir’s drops. Now, let us consider the effects of adding a form of competition to the model.

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<sup>1</sup> A possible method of improving the simulation would be going back to the calculation, and taking into account the sum that we turned into a scale factor.

### 8.3.1 Competition in the Simulation

We now have two Supply reservoirs, which compete for a given amount of consumption. The distribution of limited resources then desires to maximize the benefit to the demand, to the consumers. This models competition. Two important assumptions are made here:

1. Consumers are primarily responsible for choosing between products.
2. Competition determines how scarce resources are allocated.

First of all, we use the model to find the relationship between the production and demand heat flows, which we discussed in Chapter 5. We know that

$$V_i W_i = V_i Q_{spi} (1 - T_d/T_{si}) = \eta_i Q_{sci}, \quad (8.7)$$

where we use  $i$  as an index to identify the reservoir. We can also recall that,

$$\delta Q_{s1} = (V_1 - 1) Q_{sp1} \quad (8.8)$$

The heat flowing from the demand reservoir, which should be approximate to consumer spending, is of the form of equation 8.9 when we add the heat flowing between both Supply reservoirs and the Demand reservoir.

$$Q_{dc} = T_d (V_1 Q_{sp1}/T_{s1} + V_2 Q_{sp2}/T_{s2}) \quad (8.9)$$

It is important to note that the  $V$  coefficients are determined by market forces, and are not ad-hoc components used to simplify the model, they are an integral part of the model. The  $V_i$  are directly related to the price of goods. Since the price of a good depends on the equilibrium between supply and demand, it would seem as if the coefficients mostly measure variations in the demand, since the supply side of the problem is also in the model thanks to the thermodynamic efficiency and the amount of resources spent on production,  $Q_{sp}$ . Therefore, we can say that the  $V_i$  are determined mainly by the demand. The exact form of calculating the coefficients

theoretically is unclear, however, using the ratio of profit to investment, we can find them empirically using equation 8.8, and solving for  $V$ , equation 8.10.

Also, the implicit assumption is that Demand wishes to maximize consumer spending. We do not consider if the objective is actually minimizing  $T_d$ . This is another topic for further study. We shall continue with the assumption that Demand maximizes its utility, and that utility is equivalent to the sum of the heat energy exiting the demand reservoir,  $Q_{dc} = Q_{dc1} + Q_{dc2}$ .

$$V_1 = \frac{\delta Q_{s1}}{Q_{sp1}} + 1 \quad (8.10)$$

Then let us force the sum of the heats flowing to the work production to be  $Q_{sp} = Q_{sp1} + Q_{sp2}$ , which is equivalent to the budget restraint used in economics, which states how much money can be spent by the economy in total. This also simplifies the calculations involved in the model drastically. This begs the question of how to allocate that limited amount of resources. The answer comes if we assume that the distribution maximizes  $Q_{dc}$  as defined in equation 8.9. Maximizing that equation in terms of  $Q_{sp1}$  and  $Q_{sp2}$  involves taking the first derivative with respect to  $Q_{sp1}$  and setting it equal to zero. We note that in order to satisfy the budget constraint,  $\frac{dQ_{sp1}}{dQ_{sp2}} = -1$ . The result is equation 8.11, ignoring the partial derivatives of the temperatures and the  $V$  coefficients with respect to  $Q_{sp1}$ .

$$\frac{dQ_{dc}}{dQ_{sp1}} = T_d(V_1/T_{s1} - V_2/T_{s2}) + \dots = 0 \quad (8.11)$$

It is crucial to recognize that in order to have that point be a maximum, its second derivative must be negative. The derivative can be zero, but it is just as possible for it to be just a saddle point, maybe even a minimum. Indeed, it is even possible that equilibrium points are not stable. This problem further complicates our analysis. The second derivative must be negative, and that negative sign has to come from the

partial derivatives we ignored in equation 8.11, because otherwise the second derivative will always be zero, which does not help at all. That is an important constraint, especially if we assume that the temperature changes effected by  $Q_{sp1}$  and  $Q_{sp2}$  are small with respect to the total heat contained in the reservoirs. In other words, expenditures are not large compared to total capital, so that the temperature of the system will not change appreciably. This is logical, because it is fairly obvious that the amount of resources available do not fluctuate dramatically in stable systems, which are the only ones we consider in this version of the model. Therefore, it is a very reasonable step to only take into account the  $V$  coefficient partial derivatives, ignoring the temperature partial derivatives in the total derivative, as in equation 8.12<sup>2</sup>.

$$\frac{dQ_{dc}}{dQ_{sp1}} = T_d(V_1/T_{s1} - V_2/T_{s2}) + T_d\left(\frac{\partial V_1}{\partial Q_{sp1}} * Q_{sp1}/T_{s1} + \frac{\partial V_1}{\partial Q_{sp1}} * Q_{sp2}/T_{s2}\right) = 0 \quad (8.12)$$

Then, in order for the total derivative to be a maximum, we force the second total derivative to be smaller than zero. That result is equation 8.13

$$\frac{d^2Q_{dc}}{(dQ_{sp1})^2} = T_d\left(\frac{\partial^2 V_1}{(\partial Q_{sp1})^2} * Q_{sp1}/T_{s1} + \frac{\partial^2 V_1}{(\partial Q_{sp1})^2} * Q_{sp2}/T_{s2}\right) + 2\left(\frac{T_d}{T_{s1}} \frac{\partial V_1}{\partial Q_{sp1}} - \frac{T_d}{T_{s2}} \frac{\partial V_2}{\partial Q_{sp1}}\right) \quad (8.13)$$

The sign of the second derivative will be determined by the sign of the  $V$  coefficient partial derivatives, because generally the larger the amount of goods produced, the smaller the benefit due to their consumption will be, due to diminishing returns to consumption. Empirical use of equation 8.13 will have to wait for use of actual data and direct comparison with standard economics.

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<sup>2</sup> In physical terms, this means we assume that the temperatures are independent of the  $Q_{sp}$ . In other words, the amount of money flowing to and from the reservoirs does not strongly affect their temperature during the cycle. After the fact is another story.

In fact, as we have observed before, there are substantial parallels to the indifference curves in economics. Indifference curves are basically a way to express in graphical terms the decision making process for consumers. The consumers must choose between two goods (or a specific good competing with all other goods), maximizing their utility. However, the amount of money is not infinite, so they are constrained by a budget, a maximum amount of spending. In that case, the budget constraint is defined as before, and the indifference curves are defined by the constant  $Q_d$  curves in a  $Q_{s1}$  vs.  $Q_{s2}$  graph. And the process of identifying the maximum  $Q_d$  compatible with the budget constraint is the same as finding that maximum as conventionally done in economics textbooks[6]. In fact, the requirement of the negative sign of the second derivative of  $Q_{dc}$  is mathematically equal to the requirement that the indifference curves be convex. This is an important finding because indifference curves are the backbone of price theory. Having said all this, we turn to the simulation itself.

### 8.3.2 Simulation Results

For our simulation attempt, we assume equilibrium in  $Q_{dc}$ . We take the total derivative with respect to the spending by one of the supply reservoirs. In this case, we postulate a total amount of supply spending that is fixed. However, we can assume that the Value Multiplier of each of the reservoirs (where each of the reservoirs represent a particular good) does not depend on the amount of the good consumed. This forces us to use as a proxy for the Value Multiplier  $V_2/T_{s2} = V_1/T_{s1}$ , which is a necessary result of making the derivative of  $Q_{dc}$  equal zero according to equation 8.11. We must note that this makes it impossible for the model to prioritize one form of spending over another. If the ratio of the Value Multiplier depends on the ratio of the temperatures, then that would mean that we ignore the characteristics of the good and merely assign more utility to the more efficiently produced good. Furthermore, that assumption also makes it impossible to have

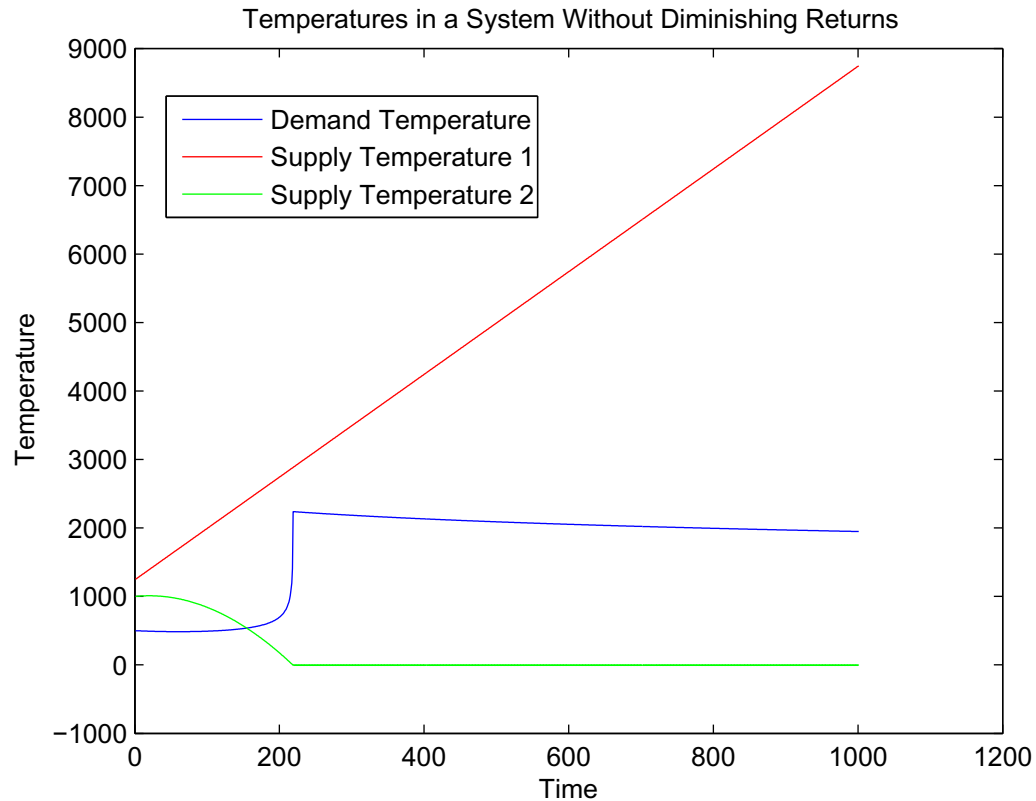


Figure 8–2: This is the Behavior of the Reservoirs when the Value Multiplier is considered, but without considering Diminishing Returns

a maximum in  $Q_{dc}$ , because the second derivative quickly converges to zero when the Value Multiplier does not have a peak. Nevertheless, we proceed after having allowed for these shortcomings in the simulation, and the results are shown in Figure 8–2. We can make a few remarks upon observation of that result. First, the more efficient supply reservoir makes the less efficient one collapse. The demand reservoir stops spending on it and takes it over, but the supply reservoir then takes back that wealth. We hesitate to read too much into a toy model such as this one, but we present the result in hopes of sparking further interest.

However, we must now address once more areas where our simulation could be improved.

Competition is part of the Value Multiplier concept. Even without directly accommodating diminishing returns, it follows that competition between two reservoirs makes the better one dominant. Value Multiplier rewards productivity and efficiency. However, in real life, diminishing returns would somewhat check that tendency towards total monopoly. Without it, one supply reservoir would quickly engulf the others, thus eliminating the free market.

We have assumed that the amount of consumption of a good does not affect the Value Multiplier. But that is not true, because there is only so much we can do with a certain product. However, it is useful as the utility would not have a peak and it allows us to consider what would happen if there was no limit to our taste for an item. If the Value Multiplier of a good depended on the amount of production, it would allow us to have a true maximum spending point. In addition it would permit us to calculate the relative supply spending that would optimize consumption. Any sketch of the Value Multiplier function would have to take into account diminishing returns. The model has assumed constant Value Multiplier, ignoring that it must have a peak value to guarantee a maximum in an equilibrium situation (indeed, without a nonzero second derivative, true equilibrium is mathematically impossible). However, that is not a realistic assumption. Diminishing returns reflects the limits imposed by human taste on unrestrained consumption. It is a reflection of the bargaining power of the demand. If it does not wish to consume uncontrollably, it can retain some of the productivity gains reflected in the raw power of the Value Multiplier. A possibility to improve the toy model could be using utility models from Economics in our calculation of the Value Multiplier.

Diminishing returns, therefore, can lessen the 'natural' behavior of the supply reservoirs have of stripping the demand and trying to become monopolistic. Therefore, unrestrained consumption can be viewed as a surrender of the bargaining power of the demand before the supply. In a sense, economic growth is directly related to

productivity growth. But that growth is distributed based on the relative bargaining powers of the reservoirs, reflected in the Value Multiplier. The model attempts to simplify such theoretical investigations within a well understood physical system.

Also, up till now, we have optimized supply spending. However, we can also optimize demand spending. This allows us to define price elasticity and cross elasticity. For example, inspection of the derivatives of Demand Spending with respect to changes in the Value Multiplier of the several reservoirs could allow us to make direct analogies to price elasticity of demand and cross elasticity: the changes in demand due to relative price and utility changes.

#### **8.4 Some Final Thoughts**

One factor we have not taken into account fully is time: deferred consumption or loans for investment. When taking into account time, it is necessary to consider savings. Savings allow the utility of consuming something now against saving the money and consuming later. A similar relationship exists to lending. To consider adequately this problem, more study into the role of the banking system will be necessary. For now, we have assumed that all that is consumed is produced. In a similar vein, the role of banking as a way to turn money that could be consumed by the demand into investment for the supply also merits greater examination. Since most businesses use loans and investment by others to begin production, the role of banking has to be evaluated in a compatible way

The role of government is also crucial. It can act as a regulator that changes the Value Multiplier coefficients, and can also redistribute income from one reservoir to the other. Retailers, a critical middleman between production and consumption, are important as well. The effect, for example, of a Wal-Mart on the global economy cannot be ignored.

In closing, we have presented in this chapter some of the results we have achieved, and avenues of further research. In particular, our conclusions are:



- There are parallels between the relationship that links micro and macroeconomics and the relationship between statistical mechanics and thermodynamics.
- Heat engines provide a mechanism for making such an analogy.
- The Value Multiplier adds an essential human element to any such physical model.
- Stimulus spending is an area where the model seems to find a link to reality, in particular in assessing the effect of the macroeconomic stimulus on a microeconomic model.
- Simulations of the model are possible, but they must take into account the difficulty in defining heat capacity.

We hope that in the future, more work is completed here, in this field where physics and economics may meet.

# APPENDICES

## APPENDIX A

# BASIC PHYSICAL CONCEPTS

What is thermodynamics? To truly answer that question and provide a primer on the basic physical theory behind the model, we must go into the fundamentals of physics. Physics in general is the study of energy in its different manifestations, and thermodynamics in particular deals with the concept of thermal energy. The central concept of thermodynamics is temperature.

Temperature is defined in abstract terms by what we call the Zeroth Law of Thermodynamics. Thermal equilibrium occurs when there is no net energy flow between two bodies. If two bodies are at thermal equilibrium, then they have the same temperature. Once we go into the physics of the concept and its application to real systems, temperature reflects the energy content of the average particle. The more temperature, the 'faster' the particles that make up a physical body are moving and the more energy they contain. Of course, wherever a physicist or an engineer sees energy, he attempts to harness it. In order to use that energy, it must be converted from its thermal form to something we call work.

Work is basically the kind of energy that can move a body in a single, defined direction. Mathematically, work is the product of the force applied and the movement in the direction specified. When we talked about thermal energy, we said it was related to the movement of the particles that form a macroscopic body. However, that movement is directed randomly. This movement can exert a force, but that force is equally directed in every direction, so no single direction can be defined.

We saw that to be able to talk about work, we must be able to define a direction of movement. So the movement of the particles due to thermal energy must be organized. The degree to which a system is disorganized is measured by the entropy,  $S$ . The entropy measures the amount of different ways a system can be organized.

To organize a systems takes energy. The more entropy a body has, the more energy it takes to order it. Indeed, in systems where the entropy can be found as a function of the thermal energy, we can define temperature as

$$\frac{1}{T} = \frac{dS}{dE}. \quad (\text{A.1})$$

Therefore, the lower the entropy, the more work can be extracted. To turn thermal energy into work, however, is not trivial. It is accomplished by means of a Heat Engine.

The Heat Engine consists of two reservoirs of thermal energy with a given temperature. Particles with a temperature  $T_h$ , and a corresponding thermal energy are all used to do work. However, only a fraction of the energy can be extracted, even under perfect conditions, as a consequence of the Second Law of Thermodynamics. It says that the total amount of entropy in an self-contained<sup>1</sup> system must either stay the same or increase. Therefore, there is a theoretical limit to the efficiency of the heat engine. Since energy is conserved, the part of the energy that does not become work is deposited in the cold reservoir, with temperature  $T_c$ .

Let us take a heat flow of  $Q_H$  out of the hot reservoir. The system produces an amount of work,  $W$ , and the rest is sent to the cold reservoir,  $Q_C$ . The efficiency is defined as  $\eta = W/Q_H$ , the fraction of energy that becomes work. However, due to

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<sup>1</sup> In other words, an adiabatic system with no energy flow in or out

conservation of energy,  $Q_H = W + Q_C$ , so

$$\eta = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}. \quad (\text{A.2})$$

In our model, we assume that the heat engine functions in accord to the Carnot cycle, the most efficient cycle theoretically possible. The characteristics of that cycle demand that  $\frac{Q_C}{Q_H} = \frac{T_C}{T_H}$ . Therefore, the efficiency equals

$$\eta = 1 - \frac{T_C}{T_H}. \quad (\text{A.3})$$

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